Asset Pricing with Consumption Frictions*

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Abstract

We study asset pricing with consumption frictions. Frictions in consumption include adjustment costs which prevent a consumer from adjusting consumption freely, due to transaction costs, commitments, search and learning costs, and psychological costs. The stochastic discount factor is determined by a marginal investor's marginal utility of adjustable consumption, and has the same form as that of the external habit model, similarly to the models with stock market non-participation. We next identify a potentially important source of risk for asset pricing: simultaneous adjustments of frictional consumption by a large number of population in the economy, which can have a large impact on interest rates. This risk is present neither in the habit model nor in other models, and provides another reason why governments and central banks are much concerned about households' consumption. However, the risk does not affect the market price of risk. We investigate the long-term behavior of asset prices, and show that equity strips have no excess volatility over zero-coupon bonds in the long run, if the stochastic discount factor admits the Hansen-Scheinkman decomposition with appropriate conditions.

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1 Introduction

We study asset pricing in the presence of consumption frictions. Frictions in consumption include adjustment costs which prevent a consumer from adjusting consumption freely, due to transaction costs, which are prominent for durable goods (Grossman and Laroque, 1990), commitments (Chetty and Szeidl, 2007, 2016), search and learning costs (Sims (2003), Brunnermeier (2004), Reis (2006), and Tutino (2013)), and psychological costs (Kahneman and Tversky (1979), and Tversky and Kahneman (1992)). Commitments alone comprise close to 65% of household consumption (Chetty and Szeidl, 2007), and hence, the share of consumption subject to frictions, which we call *frictional consumption*, in total consumption is larger. Consumption frictions also have an extra explanatory power over existing models for recent empirical findings on individual consumption (Fuster et al., 2020). Our objective is to understand the implications for asset pricing of the significant share of frictional consumption.

We study a continuous-time exchange economy in which agents' consumption consists of adjustable consumption and frictional consumption, and their preferences are represented by additively separable functions of two types of consumption. The stochastic discount factor (SDF) is determined by a marginal investor's marginal utility of adjustable consumption. Adjustment costs introduce a wedge between the investor's marginal utility of adjustable consumption and that of frictional consumption. Accordingly, the SDF is determined solely by the former, and has the same form as that of the external habit model of Campbell and Cochrane (1999). The result is similar to the finding that the coexistence of stock market participants and non-participants results in an SDF essentially of the same form as in the habit model: non-participants' consumption does not enter the marginal utility of a marginal investor (Basak and Cuoco (1998), Gomez and Michaelides (2008), Guvenen (2009) and Panageas (2020)). In this sense, non-participants' consumption can also be regarded as frictional consumption: non-participation causes a wedge between the marginal utility of non-participants' consumption and that of participants' consumption. Besides non-participants' consumption, we provide another important source, *consumption subject to adjustment costs*, which contributes to the identification of the habit stock, that is not easily identifiable. Chetty and Szeidl (2016) note the equivalence of the utility function of a representative agent in an economy with commitments with that in the seminal paper by Campbell and Cochrane (1999). However, they do not investigate the asset pricing implications of this equivalence.

We next identify a potentially important source of risk for asset pricing: *simultaneous adjustments of frictional consumption* by a large proportion of population in the economy. This risk is present neither in the habit model nor in other models. Even in the absence of any substantial change in aggregate output, interest rates move dramatically, if people adjust frictional consumption simultaneously; they move upward if people adjust consumption downward, and move downward if people adjust consumption upward. Although absent in our simple exchange economy, large detrimental effects of the sudden interest rate hike can exist in the real world, as we saw from the recession during 1979-1981 and from the experience of Asian countries during the Asian financial crisis over 1997-1998. This provides a reason why governments and central banks are much concerned about households' consumption. The recent relief payments and subsidies by governments and quantitative easing by central banks, after the onset of the COVID19 pandemic, can be thought of as means to prevent simultaneous downward adjustments of frictional consumption from happening. A similar large negative impact on interest rates caused by liquidity constraints has been noted by Detemple and Serrat (2003). In our model with consumption frictions, a positive large impact on interest rates is also possible, when people simultaneously adjust frictional consumption downward, whereas it is absent in their model. Large increase in interest rates seems to be more of a concern to policy makers than large decrease, particularly since the Great Recession (Brunnermeier et al., 2021). However, the simultaneous adjustments of frictional consumption do not affect the market price of risk, and they have effects on asset prices only though interest rates. The result is similar to the finding that liquidity constraints do not affect the equity premium by Detemple and Serrat (2003).

The above feature of our consumption-friction model is related to the rare disaster model (Rietz (1988), Barro (2006), Gabaix (2012), and Wachter (2013)), in which a looming disastrous event has large effects on asset prices. In the consumption-friction model the rare event is simultaneous adjustments of frictional consumption by a large group of households and arises endogenously, whereas the rare event is an exogenous output shock in the rare disaster model. In this sense we endogenize the rare event as that driven by demand shocks and link the rare disaster model to the habit and non-participation models.

We also investigate the long term behavior of asset prices in consumption-friction models by using the Hansen-Scheinkman (HS) decomposition (Alvarez and Jermann (2005), Hansen et al. (2008), Hansen and Scheinkman (2009), Borovicka et al. (2011), Barberis et al. (2001), and Qin and Linetsky (2017)). The SDF is decomposed into three components: a deterministic growth factor, a stationary component, and a martingale part. By using the Malliavin calculus, we show, in particular, that there is essentially no excess volatility and no high equity premium of equity strips over zero-coupon bonds in the long run, if the SDF admits the HS decomposition with appropriate conditions. Campbell and Cochrane's model does not admit such decomposition, and our result is a formal justification of their statement that non-stationarity of a power of their surplus consumption ratio is necessary for the equity premium to remain high in long horizons. Other consumption-friction models, e.g., our basic model and the models by Menzly et al. (2004) and Santos and Veronesi (2010), admit the decomposition and thus there is no long-run equity premium.[1]

An ingredient in our approach is to introduce the costs of adjustment into the utility function as utility costs, not into wealth dynamics as pecuniary costs, in contrast to the previous research

[1]See Borovicka et al. (2011) for discussion of the long-term behaviors of models by Campbell and Cochrane (1999) and Santos and Veronesi (2010).

(cf., Grossman and Laroque (1990), Flavin and Nakagawa (2008), and Chetty and Szeidl (2016)). Agents' consumption policy is an (S, s) policy, involving inaction, similarly to the previous research.[2] However, one consequence of our modeling approach is tractability, since it does not introduce frictions in the financial market nor in the goods market. We obtain equilibrium consumption policies and asset prices in closed form in our basic model, whereas policies are obtained numerically and equilibrium asset prices have not been obtained in the previous research.[3] The utility function can also be interpreted as that of loss aversion toward consumption changes, as we show in Appendix A.

Our model can also be interpreted as a heterogeneous agent economy with heterogeneous loss aversion. The interpretation relies on the re-interpretation of our utility function as that of loss aversion toward consumption changes: there are non-loss-averse agents, who do not exhibit loss aversion toward consumption changes, and loss averse agents, who exhibit loss aversion toward consumption changes. The former's consumption appears to be freely adjustable and the latter's consumption appears to be sticky. Recent research has shown evidence for wide-ranging heterogeneity in consumption behavior. Although the average MPC (Marginal Propensity to Consume) to income shocks is positive, a significant proportion of households does not adjust or expresses unwillingness to adjust consumption: the estimated proportions vary from more than 50% up to 90% (Misra and Surico (2014), Jappelli and Pistaferri (2014), Bunn et al. (2018), Christelis et al. (2019), and Fuster et al. (2020)). The heterogeneity is also observed in the timing and amount of consumption changes; a certain group of households adjusts consumption more quickly and by larger amount than do other groups in response to economic shocks (Arellano et al. (2017), and Giorgi and Gambetti (2017)). Consequently, consumption of loss averse consumers also contributes to the share of frictional consumption in total consumption.

We first study a basic model in which the adjustment of frictional consumption is simultaneous, i.e., all the agents adjust simultaneously frictional consumption, and obtain equilibrium policies and asset prices in closed-form. We next extend the basic model and study the effects of non-simultaneous adjustments of frictional consumption on asset pricing.

The linkage between households' consumption choice and asset prices has been a central theme in macroeconomics and finance. The consumption-based capital asset pricing model(CCAPM) is an avenue providing the linkage (Breeden (1979), Grossman and Shiller (1982), Lettau and Ludvigson (2001), Parker and Vissing-Jorgensen (2009), and Panageas (2020)). As we mentioned, frictions in consumption cause a wedge in the marginal utilities: the marginal utility of freely adjustable consumption and that of consumption subject to frictions are different. This is why

[2]In our model agents adjust frictional consumption to the nearest boundary of the inaction interval, whereas they adjust it to a third point in the inaction interval in the previous works. The difference comes from the nature of the models; in our model the costs are proportional to changes, but they are proportional to the values of goods in the previous research.

[3]We show in our companion paper that such a model admits a closed-form solution to a consumption and portfolio choice problem (Choi et al., 2021).

Grossman and Laroque (1990), who studied durable goods subject to transaction costs, refuted the CCAPM. Since their seminal work there have been continuous efforts to study the dynamics of frictional consumption. The study of irreversible purchase of durable goods by Hindy and Huang (1992, 1993), of irreversible decision to increase consumption by Dybvig (1995), of housing adjustments by Flavin and Nakagawa (2008), and of consumption commitments by Chetty and Szeidl (2007, 2016) are prominent examples of such endeavor. Despite the important contributions, there is no systematic analysis of equilibrium asset prices in the presence of consumption frictions.[4] Although Grossman and Laroque obtained the capital asset pricing model(CAPM) and Flavin and Nakagawa resurrected the CCAPM by introducing freely adjustable consumption, their models are based on exogenously given asset prices. Furthermore, although there exists currently highly intensive research on asset pricing with institutional frictions (Brunnermeier and Sannikov (2014), Brunnermeier et al. (2011), Krishnamurthy and He (2013), Koijen and Yogo (2019), and Brunnermeier et al. (2021)), there is relatively little endeavor to understand asset pricing with consumption frictions. In this paper, we take a first step toward obtaining an equilibrium with endogenously determined asset prices in the presence of consumption frictions.

Our study helps to distinguish the contribution of the external habit model from that of the rare disaster models and the long-run risk models (Bansal and Yaron (2004) and Bansal et al. (2012)). The habit model is a result largely of consumption frictions, including adjustments costs for durable goods and commitments, loss aversion of consumers, and non-participation in the stock market. Consequently, it is an outcome of physical heterogeneity of goods and psychological or learning-related heterogeneity of inhabitants in the economy. In contrast, the long-run risk and rare disaster models capture risks in outputs. Accordingly, there is no inconsistency in these classes of models. All the models can be regarded as complementary, and their combination will help understand the nature of macroeconomy and asset pricing.[5]

In addition to the works mentioned above, there is a handful of literature related to our work. Lettau and Wachter (2007, 2011) study the term structure of risk and return trade-off with exogenously specified SDFs. Binsbergen et al. (2012, 2017) and Gormsen (2020) study the term structure of risk and return by using equity strips. In this paper, we derive the long-term behavior of asst prices theoretically by using the HS decomposition. Barberis et al. (2001) study asset pricing in the presence of loss aversion toward wealth changes. Our model can be viewed as that of agents with loss aversion toward consumption changes. Wang (1996), Basak and Cuoco (1998), Chan and Kogan (2002), Dumas et al. (2009), Bhamra and Uppal (2009, 2014), Longstaff and Wang (2012), and Gârleanu and Panageas (2015) investigate models with heterogeneous agents. We also consider consumer heterogeneity and focus on heterogeneity in consumption adjustments

[4] Grossman and Laroque (1990) says on p. 45, "We have not met with success in solving a general equilibrium model where the assets are in a fixed supply, and a production function is specified for durable goods, so that the price process is endogenous."

[5] Santos and Veronesi (2010), Chen et al. (2011) and Barro and Jin (2021) are works in this direction.

or in loss aversion, which none of these works investigate. For recent surveys of asset pricing we refer to Cochrane (2017), Panageas (2020), and Brunnermeier et al. (2021).

The rest of the paper is organized as follows. In Section 2 we introduce the basic model. In Section 3, we describe how business cycles are generated in our model and characterize the consumption dynamics along a business cycle. Section 4 discusses asset pricing in the basic model, and Section 5 explains the extension of the basic model. Section 6 studies long-term behavior of asset prices. In Section 7 we conclude. All the proofs are provided in the Appendix.

2 Basic Model

2.1 Pure Exchange Economy

We consider a pure exchange economy endowed with an amount $y_t dt$ of outputs over the infinitesimal time period $[t, t+dt)$. The aggregate endowment (output) process y_t evolves according to the dynamics

$$\frac{dy_t}{y_t} = \mu dt + \sigma dB_t, \tag{1}$$

where B_t is a standard Brownian motion on a standard probability space $(\Omega, \mathcal{G}, \mathbb{P})$ endowed with an augmented filtration $\{\mathcal{G}_t\}_{t\geq 0}$ generated by the Brownian motion B_t. Both μ and σ are positive constants.

The model we present below is a representative agent economy with heterogeneous goods with different adjustment costs, one freely adjustable good and the other good subject to adjustment costs. The second good describes a broad category of consumption goods whose consumption requires adjustment costs such as information or learning costs. We will call the first good the *adjustable good*, its consumption *adjustable consumption*, and the second good the *frictional good*, its consumption *frictional consumption*. We will use the abbreviation: A-good, A-consumption, F-good, and F-consumption. The endowment can be used to produce the A-good and the F-good. The production is such that one unit of the endowment can produce one unit of the A-good or one unit of the F-good. The endowment and goods are perishable and cannot be stored. The production technology of the A- and F-goods implies that the equilibrium relative price between the goods is one.

There is a large number of identical agents, who are represented by a representative agent. The agent has the following utility function:

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{w\left(\left(u(c_{F,t})dt - \alpha du_t^+ - \beta du_t^-\right)\right) + (1-w)u(c_{A,t})dt\right\}\right], \tag{2}$$

where $c_{F,t}$ denotes consumption of the F-good and $c_{A,t}$ denotes that of the A-good, w $(0 \leq w \leq 1)$ is the utility weight of the F-good and $1-w$ is that of the A-good, and α and β are constants such that $\frac{1}{\delta} > \alpha \geq 0, \beta > 0$, denoting the proportional utility costs of adjustment. The term

αdu_t^+ is the utility cost of increasing frictional consumption, proportional to the increase du_t^+, and βdu_t^- is the utility cost of decreasing frictional consumption, proportional to the decrease du_t^-. Formally, u^+ and u^-, respectively, denote the positive and negative variations of the process $u(c_{F,t})$. Brunnermeier (2004) shows that in a one-period multi-good economy, learning to re-optimize in accordance with a change in income results in adjustment costs. Utility costs can also be derived from costs of acquiring and processing information in the rational attention models (Sims (2003), Reis (2006), and Tutino (2013)). In this sense, the utility function can be regarded as a reduced-form of an intertemporal utility function with learning or information costs. The utility function can also accommodate monetary transaction costs, if agents reflect the present utility value of the costs in their utility functions. We provide a detailed explanation of the utility function in Appendix A. In the Appendix we define the intertemporal loss aversion (ILA) L for F-consumption by

$$L \equiv \frac{1+\delta\beta}{1-\delta\alpha}. \tag{3}$$

By our assumption $L > 1$, and we can regard the agent as exhibiting loss aversion toward changes in frictional consumption, as we show in the Appendix. The utility functions with two different sets of parameters (w_1, α_1, β_1) and (w_2, α_2, β_2) are ordinarily equivalent if the two parameter sets induce the same ILA and the following condition is satisfied, by (47) in Appendix A:

$$\frac{w_1(1-\delta\alpha_1)}{1-w_1} = \frac{w_2(1-\delta\alpha_2)}{1-w_2}. \tag{4}$$

We will assume that the felicity function takes the following form for both goods:

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma}}{1-\gamma}, & \gamma > 0,\ \gamma \neq 1, \\ \log c, & \gamma = 1. \end{cases} \tag{5}$$

That is, the representative agent has constant relative risk aversion with the same coefficient of relative risk aversion (CRRA) γ towards risk affecting both goods.

The model can also be interpreted as that of a single-good heterogeneous agent economy, in which two classes of identical agents represented by agent L (a loss-averse agent) and agent N (a non-loss-averse agent). We provide the interpretation in Appendix B.

There is a financial market in which claims to the future endowment flow and instantaneous investment vehicles are traded. The claim to the full endowment flow is called *equity* or *stock*. The total number of shares of the stock is 1. The instantaneous investment vehicles are called (instantaneous) *bonds*, and if one invests one unit of output at time t in the bonds one gets $1+dR_{f,t}$ at time $t+dt$. Financial claims are traded with no friction, i.e. there exist neither transaction cost nor short-selling restriction on bonds and stocks. The bonds are in zero net supply. The agent is endowed with one unit of the share and zero unit of bonds at time 0. Since there exists only one source of risk, with no trading friction, the financial market is dynamically complete, i.e., all contingent claims to the economy's future endowment can be replicated by trading financial assets (Duffie and Huang, 1985).

We denote the price of a share of the stock by S_t and normalize the price of a bond to be 1. We denote the number of shares and bonds owned by the agent at time $t \geq 0$ by N_t^S and N_t^B, respectively. The agent's wealth W_t satisfies the dynamics:

$$dW_t = N_t^S(y_t dt + dS_t) + N_t^B dR_{f,t} - (c_{A,t} + c_{F,t})dt, \qquad W_0 = S_0. \tag{6}$$

We make the following assumption of the growth condition that implies the agent's expected utility is uniformly bounded, given the aggregate endowment process (1).

Assumption 1.

$$\delta > \max\left(0, (1-\gamma)(\mu - \frac{1}{2}\gamma\sigma^2)\right).$$

We consider the agent's problem.

Problem 1. *Given $c_{F,0-} = c$, $y_0 = y > c$, the agent's problem is to maximize the utility function given by (2):*

$$V(y,c) = \sup_{(c_F, c_A)} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{w\left(u(c_{F,t})dt - \alpha du_t^+ - \beta du_t^-\right) + (1-w)u(c_{A,t})dt\right\}\right], \tag{7}$$

where $c_A = (c_{A,t})$, $c_F = (c_{F,t})$ are $\mathcal{G}-$ adapted processes, subject to wealth evolution in (6) and

$$c_{F,t} + c_{A,t} \leq N_t^S y_t - N_t^B, \quad \text{for all } t \geq 0. \tag{8}$$

We define the competitive equilibrium of the economy, following the definition by Radner (1972).

Definition 1 (Market equilibrium)**.**
A competitive equilibrium of the economy is the pair of price process $(S_t, R_{f,t})$ and consumption-portfolio policies $(c_{i,t}, N_t^S, N_t^B)$, $i = F, A$, such that

(i) the policies solve the agent's utility maximization problem (Problem 1),

(ii) the market clearing condition for the stock and bonds:

$$N_t^S = 1, \quad N_t^B = 0 \qquad \text{for all } t \geq 0. \tag{9}$$

We obtain the competitive equilibrium by using a standard approach. We first consider the Pareto optimal allocation. We next consider the Arrow-Debreu economy, and finally obtain the competitive equilibrium.

2.2 Pareto Optimal Allocation

We consider the planner's problem in order to obtain the Pareto optimal allocation.

Problem 2. *Given $c_{F,0-} = c$, $y_0 = y > c$, the planner's problem is to maximize the utility function given by* (2)*:*

$$V(y,c) = \sup_{(c_F, c_A)} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{w\left(u(c_{F,t})dt - \alpha du_t^+ - \beta du_t^-\right) + (1-w)u(c_{A,t})dt\right\}\right], \tag{10}$$

where $c_F = (c_{F,t}), c_A = (c_{A,t})$ are $\mathcal{G}-$ adapted processes, subject to the resource constraint

$$c_{F,t} + c_{A,t} \leq y_t, \quad \text{for all } t \geq 0. \tag{11}$$

The planner selects the optimal consumption/production of the two goods. The agent incurs utility costs when the F good is adjusted, and hence the optimal policy involves inaction; F-consumption is adjusted only when there is a change in the aggregate endowment sufficiently large to compensate for the adjustment cost. In a multi-good economy, one typically derives an equilibrium by finding period-by-period optimal allocation of aggregate consumption between the goods. However, sub-optimality of continuous adjustment of F consumption implies that the period-by-period optimization cannot be applied to Problem 2. Instead, we need to study the whole intertemporal optimization of the planner, considering the possible optimality of the non-adjustment of consumption of the F-good.[6]

We now provide a solution to the planner's problem. Note that the value function of the planner, i.e., the maximum obtainable value of the welfare function, can be expressed as a function of the current aggregate endowment and the previous level of F-consumption. For convenience, we will use the simpler notation c_t for F-consumption, and thus the value function can be written as $V(y_t, c_{t-})$.

Suppose that the planner adjusts F-consumption at t by a small amount dc over an infinitesimal time period $[t, t+dt)$, then the welfare gain can be calculated as follows:

$$V(y_t, c_{t-} + dc) - V(y_t, c_{t-}) \approx V_c(y_t, c_{t-})dc, \tag{12}$$

where $V_c = \partial V / \partial c$.[7] The cost of adjustment is given by

$$\text{Adjustment Cost} = \begin{cases} w\alpha du_t^+ = w\alpha u'(c_{t-})dc & \text{if } dc > 0, \\ w\beta du_t^- = -w\beta u'(c_{t-})dc & \text{if } dc < 0. \end{cases}$$

Thus, the adjustment is optimal only when the welfare gain is greater than or equal to the cost, i.e., $V_c(y_t, c_{t-}) \geq w\alpha u'(c_{t-})$ if $dc > 0$ and $-V_c(y_t, c_{t-}) \geq w\beta u'(c_{t-})$ if $dc < 0$. Consequently, *inaction*

[6] Constantinides (1986) and Davis and Norman (1990) have done pioneering research on optimization problems with transaction costs and shown that inaction is optimal most of the time. Stokey (2009) provides an excellent explanation of economic polices involving inaction. We will adopt their approach to the planner's problem with appropriate modifications.

[7] Here we have assumed that the dual value function is continuously differentiable with respect to c; this property follows from the usual economic consideration for optimality that leads to smooth pasting and super-contact conditions (see Dumas (1991)).

is optimal, i.e., the optimal policy requires that the planner should not adjust F-consumption if $-w\beta u'(c_{t-}) < V_c(y_t, c_{t-}) < w\alpha u'(c_{t-})$.

Note that the marginal social value of F-consumption $V_c(y_t, c_{t-})$ is different from the weighted marginal utility of consumption $wu'(c_{t-})$. As indicated in equation (12) the former measures the social benefit of adjusting F-consumption, considering the total effects of the decision on the future social welfare. The latter measures utility gain for a marginal increase in F-consumption over an infinitesimal time period $[t, t + dt)$. The marginal social value $V_c(y_t, c_{t-})$ can become negative; it is negative when the previous F-consumption has become relatively high due to inaction.

When inaction is optimal, the value function satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\delta V(y, c) \;=\; \frac{\sigma^2 y^2}{2}\frac{\partial^2 V(y, c)}{\partial y^2} + \mu y \frac{\partial V(y, c)}{\partial y} + wu(c) + (1 - w)u(y - c). \tag{13}$$

The HJB equation tells us the balance in the optimization problem; the left-hand side of equation (13) is the rate of return in welfare terms required by the planner and the right-hand side is the expected rate of return as the sum of the expected change in the welfare and the instantaneous welfare to the society.

The smooth pasting condition implies

$$V_c(y_t, c_{t-}) = w\alpha u'(c_{t-}) \tag{14}$$

when the planner increases F-consumption, and

$$V_c(y_t, c_{t-}) = -w\beta u'(c_{t-}) \tag{15}$$

when the planner decreases F-consumption. The planner adjusts F-consumption upward only when the marginal social value of F-consumption reaches the high value $w\alpha u'(c_{t-})$ and adjusts downward when the marginal social value reaches the low value $-w\beta u'(c_{t-})$, and does not adjust consumption when the marginal social value is between the high and low values. Appendix C derives a closed form solution to the HJB equation and the planner's optimal policy. We summarize the result in the following proposition.

Proposition 2.1. *The optimal consumption allocation as a solution to the planner's problem is described by two positive numbers* $\underline{c}, \bar{c}, (\underline{c} < \bar{c})$ *such that frictional consumption is not adjusted if* $(y_t, c_{t-}) \in \mathbf{NR}$*, adjusted upward if* $(y_t, c_{t-}) \in \mathbf{IR}$*, and adjusted downward if* $(y_t, c_{t-}) \in \mathbf{DR}$*, where*

$$\begin{aligned} \mathbf{IR} &\equiv \{(y, c) \in \mathcal{D} \mid 0 < \frac{c}{y} \leq \underline{c}\}, \\ \mathbf{NR} &\equiv \{(y, c) \in \mathcal{D} \mid \underline{c} < \frac{c}{y} < \bar{c}\}, \\ \mathbf{DR} &\equiv \{(y, c) \in \mathcal{D} \mid \bar{c} \leq \frac{c}{y} < \infty\}, \end{aligned} \tag{16}$$

and $\mathcal{D} \equiv \{(y, c) \mid 0 < y < \infty, \;\; 0 < c < y\}$*.* **IR**, **NR**, *and* **DR** *represent increasing region, non-adjustment region, and decreasing region, respectively. The adjustment is such that the ratio* c_t/y_t *is restored to the nearest boundary of* **NR**.

The inaction interval plays a fundamental role in the model. Intuitively, we expect that as adjustment costs increase, or equivalently, the ILA increases, the interval gets wider. We also expect that as the utility weight of F-consumption increases (or the weight of loss averse agents' increases in the other interpretation of the model) the inaction interval shifts upward. We confirm that this intuition is correct in the following proposition.

Proposition 2.2 (Comparative Statics: inaction region)**.**

(a) $\bar{c}$ *increases with* L *and* $\underline{c}$ *decreases with* L*.*

(b) $\bar{c}$ *and* $\underline{c}$ *increase with* w*.*

2.3 The Arrow-Debreu Economy and Competitive Equilibrium

The Pareto-optimal allocation can be supported by the Arrow-Debreu economy. The stochastic discount factor(SDF) in the Arrow-Debreu economy is given by

$$\mathcal{H}_t = e^{-\delta t} u'(c_{A,t}) = e^{-\delta t} u'(y_t - c_{F,t}), \tag{17}$$

where $c_{F,t}$ is F-consumption and $c_{A,t} = y_t - c_{F,t}$ is A-consumption in the Pareto-optimal allocation as in the previous subsection. Every contingent claim which promises a stream (f_t) of cash flows is priced by using the stochastic discount factor (see Lemma 2, Wang (1996)):

$$p_t = \mathbb{E}\left[\frac{\mathcal{H}_s}{\mathcal{H}_t} f_s | \mathcal{F}_t\right]. \tag{18}$$

The Arrow-Debreu equilibrium can be implemented by sequential trade of the stock and bonds and results in a competitive equilibrium, as explained by Duffie and Huang (1985).

Notice that the economies with two different parameter sets, (w_1, α_1, β_1) and (w_2, α_2, β_2), have the same equilibrium if the parameters induce the same ILA and the condition (4) is satisfied.

3 Consumption Dynamics and Business Cycles

3.1 Optimal Consumption

In this section we discuss consumption dynamics and business cycles in the competitive equilibrium of the basic model. Recalling that we denote frictional consumption $c^*_{F,t}$ simply by c^*_t, we provide the dynamic representation of the optimal consumption allocation in the following proposition.

Proposition 3.1. *The optimal consumption allocation is given by*

$$c^*_t = c_{0-} + c^{*,+}_t - c^{*,-}_t, \quad c^*_{A,t} = y_t - c^*_t, \tag{19}$$

where

$$c_t^{*,+} = \max\left\{0, -c_{0-} + \sup_{s\in[0,t)} \left(c_s^{*,-} + \underline{c}y_s\right)\right\}, \quad c_t^{*,-} = \max\left\{0,\ c_{0-} + \sup_{s\in[0,t)} \left(c_s^{*,+} - \bar{c}y_s\right)\right\}, \tag{20}$$

where $\underline{c}$ and $\bar{c}$ ($\bar{c} > \underline{c} > 0$) are constants given in Proposition 2.1.

Proposition 3.1, together with Proposition 2.1, explains that the pattern of F-consumption is consistent with the three well-known puzzles in the household consumption data: the excess smoothness of consumption, its excess sensitivity, and the disappearance (or diminishing) of excess smoothness and sensitivity for a large shock (magnitude hypothesis).[8]

The next proposition gives the sensitivity of F-consumption to shocks to the aggregate endowment. The proposition is an immediate consequence of Proposition 2.1.

Proposition 3.2. *When frictional consumption is adjusted, its sensitivity to aggregate endowment shocks takes the following form:*

$$\Delta c_t = \begin{cases} \underline{c}(\Delta y_t)^+ & \text{when it is adjusted upward,} \\ -\bar{c}(\Delta y_t)^- & \text{when it is adjusted downward.} \end{cases}$$

The sensitivity of F-consumption to a positive shock in aggregate income when consumption is adjusted upward is equal to $\underline{c}$ and the sensitivity to a negative shock when consumption is adjusted downward is equal to $\bar{c}$. Since $\underline{c} < \bar{c}$, the sensitivity to a downward adjustment for F-consumption is higher than that for an upward adjustment, consistent with empirical evidence documented by Shea (1995), Bowman et al. (1999), Jappelli and Pistaferri (2014), Bunn et al. (2018), Christelis et al. (2019), and Fuster et al. (2020).

3.2 Business Cycles

We will call the share $\mathcal{F}_t$ of F-consumption in aggregate consumption the *frictional consumption share* or simply the F-share, and the share $\mathcal{A}_t$ of A-consumption in aggregate consumption *adjustable consumption share* or simply the A-share:

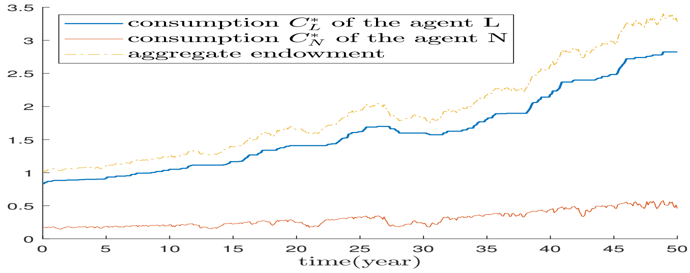
$$\mathcal{F}_t \equiv \frac{c_t}{y_t}, \quad \mathcal{A}_t = 1 - \mathcal{F}_t. \tag{21}$$

Panel (a) of Figure 1 shows simulated paths of endowment and equilibrium consumption and Panel (b) shows that of the F-share. By Proposition 3.1 the F-share satisfies

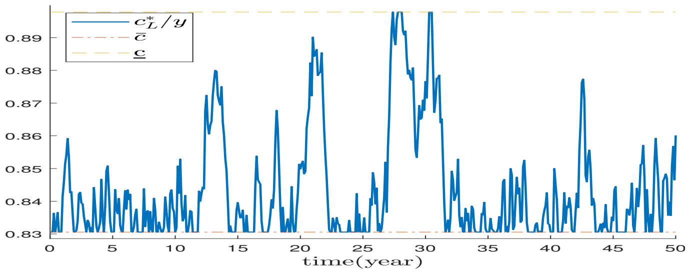
$$0 < \underline{c} \leq \mathcal{F}_t \leq \bar{c} < 1, \ \ \forall\, t \geq 0. \tag{22}$$

We call the period when the F-share is near $\underline{c}$ or during the time when $\mathcal{F}_t$ is generally decreasing a *boom*, and the period when the F-share is near $\underline{c}$ or during the time when $\mathcal{F}_t$ is generally

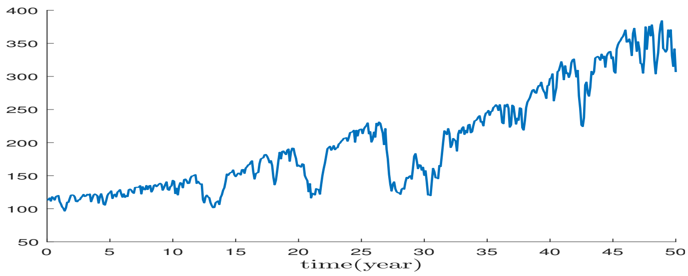
[8]Interested readers can refer to our companion paper for details (Choi et al., 2021).



(a) y_t, $c^*_{F,t}$ and $c^*_{A,t}$



(b) $\mathcal{F}_t$



(c) S_t

Figure 1: Simulation paths of y_t, $c^*_{A,t}$, $c^*_{F,t}$, $\mathcal{F}_t$, and S_t. Parameter values are as follows: $\mu = 0.023, \sigma = 0.033, \delta = 0.01, \gamma = 1, L = 1.005, w = 0.85, y_0 = 1$, and $c_{0-} = 0.8$.

increasing a *recession*. The F-share changes from $\underline{c}$ to $\bar{c}$, during which negative shocks to the aggregate endowment dominate positive shocks, and back to $\underline{c}$, during which the opposite is true. Then, the F-share displays a negative relationship with the business cycle, since F-consumption is infrequently adjusted. The share tends to be low (high, resp.) when the aggregate endowment is increasing (decreasing, resp.). Note that there is essentially no business cycle in terms of outputs, since the mean and variance of the growth rate of the aggregate endowment are constant.

A stationary distribution of the F-share exists and we give its density in closed form in Appendix E.

3.3 Elasticity of Intertemporal Substitution

In this subsection we will briefly discuss the elasticity of intertemporal substitution (EIS) of consumption of the two goods. The EIS of consumption $i = A, F$ at time t is defined by

$$EIS_{i,t} \equiv -\frac{\partial\, \mathbb{E}_t[dc_{i,t}/c_{i,t}]}{\partial\, \mathbb{E}_t[d\mathcal{H}_t/\mathcal{H}_t]}, \quad i = A, F, \tag{23}$$

where $\mathcal{H}_t$ is the SDF defined in (17).[9] Note that $-E_t[d\mathcal{H}_t/\mathcal{H}_t]$ is equal to the instantaneous bond

[9] It is not straightforward to define the EIS in the presence of risky investments and there is no single universally accepted definition (see, e.g., Vissing-Jørgensen (2002) and Dumas and Luciano (2017)). Here we adopt the conventional definition.

return for the economy at t and equation (23) is the same as the conventional definition.

We show that the EIS of A-consumption is equal to $1/\gamma$, in Appendix F, confirming the conventional wisdom that the EIS is equal to the reciprocal of the coefficient of relative risk aversion if the utility function is time separable. For F-consumption the EIS is equal to 0 when it is not adjusted. When F-consumption is adjusted, the proportional change is the same as that of A-consumption, and consequently, the EIS of F-consumption is the same as that of A-consumption. Thus, for the most part the EIS of F-consumption is equal to 0, and only at instances when it is adjusted the EIS is equal to $1/\gamma$. This is consistent with time variation and heterogeneity of EIS in the literature (Guvenen (2006) and Crossley and Low (2011)).

4 Asset Prices in the Basic Model

In this section, we derive the asset prices and investigate their dynamics over a business cycle in the basic model economy. The SDF in (17) takes the form:

$$\mathcal{H}_t = e^{-\delta t}(y_t - c_t)^{-\gamma} = e^{-\delta t}(1 - \mathcal{F}_t)^{-\gamma} y_t^{-\gamma} = e^{-\delta t}\mathcal{A}_t^{-\gamma} y_t^{-\gamma}. \tag{24}$$

The SDF is similar to that in the external habit model of Campbell and Cochrane (1999), it is the product of $y_t^{-\gamma}$, the SDF in the Lucas economy (as we show in (110) in Appendix J) and the marginal utility (negative γ-th power) of the A-share. The A-share corresponds to the surplus consumption ratio in Campbell and Cochrane (1999). The SDF in our basic model is different from that of Campbell and Cochrane in the dynamics of the state-dependent variable. In the model it is driven by the A-share which is a regulated geometric Brownian motion confined in a closed bounded interval whose left boundary is strictly larger than 0, whereas the surplus consumption ratio is confined in an open interval $(0, \mathcal{A}_{\max})$, and hence, it can get arbitrarily close to 0 in their model. Partly due to this aspect, the process $\mathcal{A}_t^{-\gamma}$ is stationary in our basic model, but it is non-stationary in their model. We will discuss the implication of stationarity when we study long-term behavior of asset prices.[10]

The price S_t of the equity(stock) at time t is given by

$$S_t = \mathbb{E}_t\left[\int_t^\infty \frac{\mathcal{H}_s}{\mathcal{H}_t} y_s ds\right], \tag{25}$$

where $\mathbb{E}_t$ denotes the conditional expectation taken at time t.

The price $P(t,T)$ of a zero coupon bond and the price P_t of a perpetuity paying at a rate equal to 1 are given by

$$P(t,T) = \mathbb{E}_t\left[\frac{\mathcal{H}_T}{\mathcal{H}_t}\right], \quad P_t = \mathbb{E}_t\left[\int_t^\infty \frac{\mathcal{H}_s}{\mathcal{H}_t} ds\right]. \tag{26}$$

We derive prices of the stock and consol bonds in closed-form and give them in Appendix G.

[10] See pp.239–240 in Campbell and Cochrane (1999) for discussion of this aspect of their model.

4.1 Sharpe Ratio and Interest Rate

From the SDF in (17), we can derive the risk-free rate and the market price of risk in terms of $\mathcal{A}_t$.

Proposition 4.1. *The market price of risk θ_t is given by*

$$\theta_t = \theta(\mathcal{A}_t) = \frac{\gamma\sigma}{\mathcal{A}_t}. \tag{27}$$

The instantaneous return $dR_{f,t}$ on bonds is given by $r_t dt + dG_t$ where

$$r_t = r(\mathcal{A}_t) = \delta + \frac{\mu\gamma}{\mathcal{A}_t} - \frac{\gamma(\gamma+1)}{2}\frac{\sigma^2}{\mathcal{A}_t^2} \tag{28}$$

and

$$dG_t = -\frac{\gamma}{y_t - c_t^*} dc_t^*, \tag{29}$$

where c_t^* is given in Proposition 3.1.

The asset returns depend on the A-share: the expected return, the instantaneous bond return, and the Sharpe ratio are all time-varying according to $\mathcal{A}_t$.

The explicit form of θ_t in (27) shows that the Sharpe ratio decreases with the A-share, $\mathcal{A}_t$. This means that the Sharpe ratio is counter-cyclical: large (small, resp.) values of $\mathcal{A}_t$ correspond to a boom (a recession, resp.).

The instantaneous bond return in (28) has two components: the first is r_t and the second is dG_t and singular, i.e., it involves dc_t^*. Note that the instantaneous bond return is equal to r_t except when F-consumption is adjusted. In general, there is a non-monotonic relationship between $\mathcal{A}_t$ and r_t, depending on the strength of the effect of growth μ of the economy and that of the precautionary motive represented by the third term in (28). The existence of the singular component dG_t implies that whenever F-consumption is increased (decreased, resp.), the instantaneous bond return decreases (increases, resp.). F-consumption increases in a boom when $\mathcal{A}_t$ hits $1-\underline{c}$, and the adjustment induces a large decrease in the instantaneous bond return, and the opposite is true in a recession. Intuitively, if F-consumption is increased (decreased, resp.), then relatively smaller (larger, resp.) proportion is available for A-consumption and the marginal utility of A-consumption increases (decreases, resp.), which lowers (heightens, resp.) the instantaneous bond return. The changes in the instantaneous bond return are infinitely large compared to the length of the time interval, thanks to the singular term dG_t in (29); the term is proportional to dc_t^*, which is of magnitude $\sqrt{dt}$ over the instantaneous time interval with length dt, and hence infinitely large compared to the time length. Accordingly, they have large effects on bond yields and the equity price. A similar large change of the instantaneous bond return occurs in a model with liquidity constraints studied by Detemple and Serrat (2003). In their model the change occurs only in the negative direction, so the large adjustment due to liquidity constraints lowers bond yields. However, in our model, both positive and negative large adjustments are possible and the bond yields can be influenced by both adjustments. The bond yields tend to be more influenced by the potential future large upward adjustments of F-consumption in a boom, and the opposite is true in a recession. Consequently the interest rates tend to be low in a boom and high in a recession. The model, thus, has a feature similar to rare events in the rare disaster models (Rietz (1988), Barro (2006), Gabaix (2012), and Wachter (2013)). That is, there is a looming potential event that a large group of agents simultaneously adjust their consumption, and has potentially large effects on the interest rates of the economy.

Notice that the adjustment of F-consumption, dc_t^* influences bond yields, but not the market price of risk. The result is similar to the result in Detemple and Serrat (2003) that liquidity constraints do not affect the equity premium.

4.2 Asset Prices and Moments

In this subsection we show results of numerical simulation. For computation we take the values of the mean and growth rate of output from Gârleanu and Panageas (2019) and set $\mu = 0.023, \sigma = 0.033$. For preference parameters we take the following values: $\delta = 0.01, \gamma = 1, L = 1.005, y_0 = 1, c_{0-} = 0.8$.

Moment	Data	Our Model					
		$w = 0.65$	$w = 0.7$	$w = 0.75$	$w = 0.8$	$w = 0.85$	$w = 0.9$
mean stock return	0.0547	0.0381	0.0402	0.0438	0.0505	**0.0654**	0.1098
return volatility	0.2017	0.0935	0.1091	0.1310	0.1639	**0.2187**	0.3303
mean risk-free rate	0.0056	0.0292	0.0282	0.0264	0.0233	**0.0166**	-0.0018
SD of risk-free rate	0.028	0.0378	0.0476	0.0615	0.0827	**0.1191**	0.1966
mean Sharpe ratio	0.2333	0.0943	0.1101	0.1321	0.1651	**0.2202**	0.3303
SD of Sharpe ratio	0.2993	0.0040	0.0058	0.0088	0.01426	**0.02579**	0.05731
mean log wealth-consumption ratio	4.68	4.17	4.25	4.32	4.38	**4.45**	4.51
SD of log wealth-consumption ratio	0.2468	0.0407	0.0501	0.0626	0.0804	**0.1076**	0.1557

Table 1: SD means standard deviation. The data for the stock return and risk-free rate are from Beeler and Campbell (2012). The data for the Sharpe ratio are from Andrea et al. (2019). The data for the log wealth-consumption are from Lustig et al. (2013). The moments are computed using the stationary density.

ξ	$\underline{c}$	25th-percentile	median	75th-percentile	$\bar{c}$
interest rate	-0.1951	-0.0746	0.0372	0.1094	0.5274

Table 2: Interest rates

Table 1 shows moments of asset returns for various values of the utility weight w of the F-good. We use 3-month yields as interest rates (risk-free rates) and compute the moments using the stationary density. As w increases, the Sharpe ratio, the mean and standard deviation of the

weight	interest rates			
	mean	SD	$\underline{c}$	$\bar{c}$
$w = 0.2$	0.0317	0.0051	0.0213	0.0506
$w = 0.3$	0.0315	0.0087	0.0138	0.0643
$w = 0.4$	0.0312	0.0136	0.0039	0.0829
$w = 0.5$	0.0307	0.0203	-0.0098	0.1095
$w = 0.6$	0.0299	0.0305	-0.0304	0.1506
$w = 0.7$	0.0282	0.0476	-0.0642	0.2216
$w = 0.8$	0.0233	0.0827	-0.1304	0.3708
$w = 0.9$	-0.0018	0.1966	-0.3210	0.8553

Table 3: Interest Rates: Moments and Extreme Values

equity returns increase. This is intuitive, as w increases, the A-share decreases and A-consumption becomes more volatile. If $w = 0.85$, the mean and standard deviation of equity returns and the mean log-consumption wealth ratio from the basic model matches well the data.

Table 2 shows interest rates for various values of F-share in the inaction interval for the same set of parameter values with $w = 0.85$. The interest rates exhibit wild behavior. They are volatile with a standard deviation of 11.9%, and too low and can become very negative and equal to -19% in a boom, when the F-share is low, and too high can become 52.7% in a recession, when the F-share is high. The wild behavior is due to simultaneous adjustments of F-consumption by all the agents in the economy. The instantaneous interest rate becomes negative infinite in an instant when the agents adjust consumption upward and positive infinite when they adjust consumption downward. The numerical computation confirms the effect of this large simultaneous adjustments. Table 3 gives the mean and standard deviation of interest rates and the values at the boundaries of the inaction interval. When w is low the standard deviation is small and the interest rates at the boundaries are moderate, however, as w increases the standard deviation increases, and the interest rates at the boundaries take more extreme values. Intuitively, as w increases, the equilibrium F-share increases, and the effect of simultaneous adjustments of F-consumption gets larger.

In the next section we consider an extension of the model in which adjustments of F-consumption are not necessarily simultaneous.

5 Extension of the Basic Model: Heterogeneous Adjustments of Frictional Consumption

We have studied the basic model and its implications for asset prices. We have shown that the model can generate counter-cyclical risk premia and a high equity premium. The interest rates in the model are volatile and can be too high in a recession and too low in a boom. The wild behavior of interest rates in the model are due to the singular term dG_t which is proportional to adjustment of F-consumption. The basic model is a representative agent economy, representing a large number of identical agents, and adjustment of F-consumption corresponds to simultaneous adjustments of all the agents, which have a drastically large effect on the SDF.

In the real world simultaneous changes in F-consumption, e.g., commitment consumption, durable consumption, and consumption of goods which have large psychological costs for adjustment, occur only rarely by a large group of consumers.[11] In this section we extend the basic model to accommodate non-simultaneous adjustments. We give an extension in Appendix H with heterogeneity in adjustment costs and idiosyncratic preference shocks.

In the extended model, each agent adjusts consumption at different instances, so the effect of an individual agent's adjustment is not large if her weight in the economy is small. If the weight of the agent is significantly large, then her adjustment can still have a large effect, despite it is less so than in the basic model. This corresponds to a real world situation in which a large number of individuals whose consumption comprises a significant portion in the economy adjusts their consumption.

Adjustable consumption is equal to the difference between the total output and the aggregate F-consumption. Then, the SDF takes the same form as in the basic model and in Campbell and Cochrane (1999):

$$\mathcal{H}_t = e^{-\delta t} \mathcal{A}_t^{-\gamma} y_t^{-\gamma}, \quad \mathcal{A}_t = 1 - \mathcal{F}_t, \tag{30}$$

where $\mathcal{A}_t$ is the A-share and $\mathcal{F}_t$ is the F-share. The dynamics of the A (or F) share completely determine the SDF and asset prices. They belong to the same class of models in which the SDF takes the form in (30). For convenience we call the class of models whose SDF has the same form as in (30) the *consumption friction (CF) class*.

Different models belonging to the CF class can generate different dynamics, but the underlying economic mechanism is essentially the same. Of course, welfare implications are quite different. Frictional consumption contributes to welfare of the economy, but the habit stock in the external habit model does not.

Suppose that the aggregate endowment follows (1), and the aggregate F-consumption $c_{F,t}$ follows the dynamics:

$$\frac{dc_{F,t}}{c_{F,t}} = \mu_{F,t} dt + \sigma_{F,t} dB_t + \Delta c_{F,t}, \tag{31}$$

where $\Delta c_{F,t}$denotes a singular adjustment or a jump in aggregate F-consumption which means a large adjustment of aggregate F-consumption such that $|\Delta c_{F,t}/dt| = \infty$. We provide a formal definition of $\Delta c_{F,t}$ in Appendix H.

Then, the Sharpe ratio and the instantaneous bond return in the extended economy are given in the following proposition.

[11] In our model goods are perishable. However, a durable good can be represented by a flow of the F-good.

Proposition 5.1. *The market price of risk θ_t is given by*

$$\theta_t = \gamma \frac{(\sigma - \sigma_{F,t}(1 - \mathcal{A}_t))}{\mathcal{A}_t}. \tag{32}$$

The instantaneous return $dR_{f,t}$ on bonds is given by $r_t dt + \Delta G_t$ where

$$r_t = \delta + \gamma \frac{(\mu - \mu_{F,t}(1 - \mathcal{A}_t))}{\mathcal{A}_t} - \frac{\gamma(\gamma + 1)}{2} \frac{(\sigma - \sigma_{F,t}(1 - \mathcal{A}_t))^2}{\mathcal{A}_t^2} \tag{33}$$

and

$$\Delta G_t = -\frac{\gamma}{c_{A,t}} \Delta c_{F,t}, \tag{34}$$

where $c_{A,t}$ is the aggregate A-consumption.

The market price of risk is affected by the A-share, $\mathcal{A}_t$, the volatility, $\sigma_{F,t}$ of the growth of F-consumption. An increase in the A-share reduces the market price of risk θ_t for the aggregate output, provided that the volatility of aggregate output growth is larger than the that of F-consumption growth. In our basic model, the volatility of F-consumption growth is constant, and hence the condition is valid.

The first component r_t in the instantaneous bond return is affected by the A-share, the mean growth rate of F-consumption, and its volatility. A high mean growth rate of F-consumption reduces r_t, and hence helps to resolve the risk-free rate puzzle (Weil, 1989). A high volatility $\sigma_{F,t}$ of the component of F-consumption growth, decreases the effect of the third term in (33), if the volatility of aggregate output growth is larger than the that of F-consumption growth. The second component ΔG_t of the instantaneous bond return is due to simultaneous adjustments of a large number of agents in the economy and has large effects on bond yields, as shown in the previous section.

In the model of Campbell and Cochrane (1999), the process of the habit stock is meticulously chosen so that r_t in (33) stays constant. As the variable $\mathcal{A}_t$ can approach arbitrarily close to 0 in their model, the Sharpe ratio in (32) can become very large, and consequently, can generate a large equity premium. However, the effect ΔG_t of large simultaneous adjustments is absent in their model. In our extended model, the effect is present. However, it is smaller than in the basic model, as only a subset of agents adjust F-consumption simultaneously. Notice that the simultaneous adjustments do not affect the market price of risk in (32). Therefore, they affect asset prices only through interest rates, as in the basic model.

By our investigation of the extended model, we can understand the nature of the external habit model. The habit model is a result largely of consumption frictions, including adjustments costs for durable goods and commitments, learning costs, loss aversion of consumers, and their non-participation in the stock market, or psychological habit as in the original formulation of the model. Consequently, it is an outcome of physical heterogeneity of goods and psychological or learning-related heterogeneity of inhabitants in the economy. In contrast, the long-run risks and rare disaster models capture risks in outputs. Accordingly, there is no inconsistency in these classes of models.

Our marginal contribution can be summarized as follows. Firstly, it provides a micro-foundation for asset pricing in an economy populated by the heterogeneous agents. A micro model can help identify the sources of asset price movements, whereas it is hard to identify the habit stock in the macro habit model. The micro model of adjustments, which we studied in this paper, attracts attention to adjustments of F-consumption. This leads to the second contribution of the model. Simultaneous adjustments of F-consumption can have large and significant effects on interest rates and prices of assets. The effect of simultaneous adjustments of F-consumption is not present in the habit model, and the presence of an event with potentially large effects helps link the habit model to the rare disaster (or event) model. In the rare disaster model the rare event is exogenously provided, but the event arises endogenously in models with adjustment costs. Thirdly, we investigate the long-term behavior of asset prices for models belonging to the CF class. There has been interest in the term structure of risk and return trade-offs and the long-term behavior of asset prices is an important componen the term structure (Lettau and Wachter (2007, 2011), Santos and Veronesi (2010), Binsbergen et al. (2012, 2017), and Gormsen (2020)). Particularly, we study the behavior in models whose SDF admits Hansen-Scheinkman decomposition, which include our basic model and models by Menzly et al. (2004) and Santos and Veronesi (2010), but do not include the Campbell and Cochrane model. This is our task in the next section.

6 Long-Term Behavior of Asset Prices

We will now study long-term behavior of asset prices. As previously, we assume that the aggregate endowment follows the geometric Brownian motion (1). We consider a dividend strip with maturity T which pays y_T at T. Its price $S(t,T)$ is given by

$$S(t,T) = \mathbb{E}_t \left[\frac{\mathcal{H}_T}{\mathcal{H}_t} y_t \right]. \tag{35}$$

We first study long-term risk and return trade-off. For the purpose, we consider the Hansen-Scheinkman (HS) decomposition (Alvarez and Jermann (2005), Hansen and Scheinkman (2009), Borovicka et al. (2011), Hansen (2012), Borovicka et al. (2016), and Qin and Linetsky (2017)):

$$\begin{aligned} \mathcal{H}_t &= e^{-\delta t}(y_t - c_t)^{-\gamma} = e^{-\delta t}\mathcal{A}_t^{-\gamma} y_t^{-\gamma} \\ &= e^{-\rho t}\mathcal{A}_t^{-\gamma} e^{-\frac{1}{2}\gamma^2\sigma^2 - \gamma\sigma B_t}, \\ &= e^{-\rho t}\frac{1}{\psi(\mathcal{A}_t)} M_{B,t}, \end{aligned} \tag{36}$$

where

$$\rho \equiv \delta + \gamma\mu - \frac{1}{2}\gamma(1+\gamma)\sigma^2,$$

c_t is the aggregate F-consumption, and

$$\psi(\mathcal{A}_t) \equiv \mathcal{A}_t^{\gamma}, \quad M_{B,t} \equiv e^{-\frac{1}{2}\gamma^2\sigma^2 - \gamma\sigma B_t}. \tag{37}$$

In the decomposition the SDF $\mathcal{H}_t$ is expressed as the product of the growth component $e^{-\rho t}$, the component $1/\psi(\mathcal{A}_t)$ and the martingale component $M_{B,t}$.

We consider a change of measure over $[0,T]$ by using $M_{B,t}$. Namely, we define another probability measure Q_B by $Q_B(\mathfrak{A}) = \mathbb{E}[M_{B,T}\mathbf{1}_{\mathfrak{A}}]$ for $\mathfrak{A} \in \mathcal{G}_T$, where $\mathbf{1}_{\mathfrak{A}}$ is the indicator function of set $\mathfrak{A}$, i.e., $\mathbf{1}_{\mathfrak{A}}(\omega) = 1$ if $\omega \in \mathfrak{A}$ and $\mathbf{1}_{\mathfrak{A}}(\omega) = 0$ otherwise. By the HS decomposition (36) we can derive

$$\begin{aligned} P(t,T) &= \mathbb{E}_t\left[\frac{\mathcal{H}_T}{\mathcal{H}_t}\right] = \mathbb{E}_t\left[\frac{e^{-\rho T}\psi(\mathcal{A}_t)}{e^{-\rho t}\psi(\mathcal{A}_T)}\frac{M_{B,T}}{M_{B,t}}\right] \\ &= \mathbb{E}_t^{Q_B}\left[\frac{e^{-\rho T}\psi(\mathcal{A}_t)}{e^{-\rho t}\psi(\mathcal{A}_T)}\right] \\ &= e^{-\rho(T-t)}\psi(\mathcal{A}_t)\mathbb{E}_t^{Q_B}\left[\frac{1}{\psi(\mathcal{A}_T)}\right]. \end{aligned} \tag{38}$$

By the Girsanov Theorem $B_t^{Q_B} \equiv B_t + \gamma\sigma t$ is a Brownian motion under Q_B. We assume that both $\mathcal{A}_t$ and $1/\psi(\mathcal{A}_t)$ are stationary and and $\mathcal{A}_t$ is ergodic with a stationary distribution ς_B under Q_B. The assumption is valid for the basic model and for models by Menzly et al. (2004) and Santos and Veronesi (2010).[12]

Proposition 6.1. *The limiting behavior of the zero coupon bond price is characterized by*

$$\lim_{T\to\infty} e^{\rho(T-t)}P(t,T) = A_B\psi(\mathcal{A}_t),$$

where A_B is a constant defined by

$$A_B \equiv \int_{\underline{c}}^{\bar{c}} \frac{1}{\psi(\eta)} d\varsigma_B(\eta).$$

Proposition 6.1 is essentially Proposition 7.1 of Hansen and Scheinkman (2009) applied to our economy. However, we cannot apply the latter directly to derive Proposition 6.1, as the process $\mathcal{A}_t$ involves a singular (local-time) component and Hansen and Scheinkman's model does not admit such a process. We obtain the decomposition directly from the SDF, not relying on their operator method.

The constant ρ is the long-term zero coupon yield. Proposition 6.1 is consistent with the result by Dybvig et al. (1996) that the long-term zero coupon yield can never fall; it is constant in our economy as in Qin and Linetsky (2017).

[12]The model by Santos and Veronesi (2010) does not exactly satisfy the assumption of geometric Brownian motion for the aggregate endowment process in (1) due to a small persistent predictable component in the process. However, Borovicka et al. (2011) provide an explanation of Santos and Veronesi's model in an earlier version which assumes a geometric Brownian motion for the aggregate endowment process.

We now study the long-term behavior of equity strips. We know

$$\begin{aligned}\mathcal{H}_t y_t &= e^{-\delta t}(y_t - c_t)^{-\gamma} y_t = e^{-\delta t}\mathcal{A}_t^{-\gamma} y_t^{1-\gamma} \\ &= e^{-\rho_s t}\mathcal{A}_t^{-\gamma} e^{-\frac{1}{2}(1-\gamma)^2\sigma^2 t + (1-\gamma)\sigma B_t} \\ &= e^{-\rho_s t}\frac{1}{\psi(\mathcal{A}_t)} M_{S,t},\end{aligned} \tag{39}$$

where

$$M_{S,t} = e^{-\frac{1}{2}(1-\gamma)^2\sigma^2 + (1-\gamma)\sigma B_t} \quad \text{and} \quad \rho_s = \delta - \gamma\mu - \frac{1}{2}\gamma(1-\gamma)\sigma^2.$$

Similarly to the above, we consider another measure Q_S on $[0, T]$ defined by $Q_S(\mathfrak{A}) = \mathbb{E}[M_{S,T}\mathbf{1}_{\mathfrak{A}}]$ for $\mathfrak{A} \in \mathcal{F}_T$. Then,

$$\begin{aligned}S(t,T) &= y_t \mathbb{E}_t\left[\frac{\mathcal{H}_T y_T}{\mathcal{H}_t y_t}\right] = y_t \mathbb{E}_t\left[\frac{e^{-\rho T}\psi(\mathcal{A}_t)}{e^{-\rho t}\psi(\mathcal{A}_T)}\frac{M_{S,T}}{M_{S,t}}\right] \\ &= y_t \mathbb{E}_t^{Q_S}\left[\frac{e^{-\rho_s T}\psi(\mathcal{A}_t)}{e^{-\rho_s t}\psi(\mathcal{A}_T)}\right] \\ &= e^{-\rho_s (T-t)} y_t \psi(\mathcal{A}_t)\mathbb{E}_t^{Q_S}\left[\frac{1}{\psi(\mathcal{A}_T)}\right].\end{aligned} \tag{40}$$

By the Girsanov Theorem $B_t^{Q_S} \equiv B_t - (1-\gamma)\sigma t$ is a Brownian motion under Q_S. We assume both $\mathcal{F}_t$ and $1/\psi(\mathcal{A}_t)$ are stationary and and $\mathcal{A}_t$ is ergodic with a stationary distribution ς_S under Q_S. The assumption is valid for the basic model and for models by Menzly et al. (2004) and Santos and Veronesi (2010).

Proposition 6.2. *The limiting behavior of the equity strip price is characterized by*

$$\lim_{T\to\infty} e^{\rho_s (T-t)} S(t,T) = A_S y_t \psi(\mathcal{A}_t),$$

where A_S is a constant defined by

$$A_S \equiv \int_{\underline{c}}^{\bar{c}} \frac{1}{\psi(\eta)} d\varsigma_s(\eta).$$

The constant ρ_s is the long-term equity yield and the difference between ρ_s and ρ is the *long-term equity premium*. The long-term equity yield is equal to that in the Lucas economy.[13] Thus, we have the following proposition.

Proposition 6.3. *The long-term equity premium ρ_{ep} is given by*

$$\rho_{ep} \equiv \rho_s - \mu - \rho = \gamma\sigma^2.$$

[13]We provide a discussion of the Lucas economy in Appendix J.

The long-term equity premium is equal to the equity premium in the Lucas economy and does not depend on particular characteristics of individual preferences. For example, it does not depend on ILA L nor on the utility weight w in our basic model.

The fact that the process $\mathcal{A}_t^{-\gamma}$ is stationary plays a crucial role in establishing Propositions 6.1 and 6.2. It guarantees the application of the ergodic theorem. The propositions are not valid in the model of Campbell and Cochrane (1999), as the process $\mathcal{A}_t^{-\gamma}$ is non-stationary. Borovicka et al. (2011) discuss the discontinuity in risk premia in the long horizon limit in Campbell and Cochrane's model.

We now study the returns, $dr^b_{t,T} \equiv dP(t,T)/P(t,T)$ on the zero coupon bond and $dr^s_{t,T} \equiv dS(t,T)/S(t,T)$ on the equity strip. We can calculate the changes in expectations in (38) and (40) over the instantaneous time interval $[t, t+dt)$ by the Malliavin calculus. By the Clark-Ocone formula (Proposition 1.3.14, Nualart (2006)):

$$d\mathbb{E}_t^Q\left[\frac{1}{\psi(\mathcal{A}_T)}\right] = \mathbb{E}_t^Q\left[D_t\left(\frac{1}{\psi(\mathcal{A}_T)}\right)\right]dB_t, \tag{41}$$

where Q is Q^B or Q^S and D_t denotes the Malliavin derivative. Intuitively, the Malliavin derivative measures the change in the random variable for a small shock at time t. If a process x_t is stationary, then we know the effect of a random shock today has a vanishing effect, i.e., $\lim_{T\to\infty} D_t(x_T) = 0$. Since $1/\psi(\mathcal{A}_T)$ is a stationary process, its Malliavin derivative vanishes in the limit and by (41) we know

$$\lim_{T\to\infty} d\mathbb{E}_t^Q\left[\frac{1}{\psi(\mathcal{A}_T)}\right] = 0. \tag{42}$$

By this and (38), we know that the return on a zero-coupon bond has risk equal to that of $\psi(\mathcal{A}_t) = 1/\mathcal{H}_t$, and by (40) the return on an equity strip has risk equal to that of $y_t\psi(\mathcal{A}_t) = y_t/\mathcal{H}_t$, in the long-term limit. That is, the risk in a long-term zero-coupon bond is governed by the change in the current SDF, and the risk in a long-term equity strip is governed by the changes in the current SDF and in the current cash flow, with all the future changes being averaged out. Furthermore, this result implies that the volatility of the equity strip return is the sum of that of the zero-coupon bond return and that of the current output growth, and there can be no excess volatility of the equity strip over the zero-coupon bond in the long-term limit.

Proposition 6.4. *In the long-term limit, there is no excess volatility of an equity strip over a zero-coupon bond if $\mathcal{A}_t$ and $1/\psi(\mathcal{A}_t)$ are stationary and $\mathcal{A}_t$ is ergodic in both probability measures.*

Proposition 6.4 is consistent with Proposition 6.3. The result is consistent with Campbell and Cochrane's statement that non-stationarity of $\mathcal{A}_t^{-\gamma}$ is essential for the equity premium to remain high at long horizons (p. 240, Campbell and Cochrane (1999)). The proposition implies that the risk, which affects the F-share, does not have an effect on the difference between volatilities of equity strips and zero-coupon bonds in the long term. However, it has effects on asset prices in the short and intermediate-term periods. Consequently, the risk can generate diverse term structures of equity risk premia, e.g., a downward sloping equity term structure if there are multiple assets with different risks in the economy, as studied by Santos and Veronesi (2010) (see also Borovicka et al. (2011)).

7 Conclusion

We have studied equilibrium in an economy with consumption frictions. We have investigated asset pricing implications and shown that the model belongs to the same class as that of external habit. It contributes to asset pricing by providing a micro-foundation to the CF (consumption-friction) class of asset pricing models, identifying one important source of potentially large effects, simultaneous adjustments of frictional consumption by large number of agents, and investigating the long-term behavior of asset prices in models belonging to the class.

Frictional consumption is shown to be sticky. There can potentially be other ways to model sticky consumption behavior in a heterogeneous agent model. For example, the literature on contracting model with limited commitment generates a sticky consumption pattern with adjustments similar to those in this paper (Grochulski and Zhang (2011), Miao and Zhang (2015), and Choi et al. (2020b)). An extension of the models with multiple heterogeneous agents may provide another mechanism for infrequent adjustments.

The model in this paper has only one source for driving productivity. We have abstracted from other important aspects of the real world for parsimony, e.g., production, labor income, a persistent component or long-run risk in consumption growth, changing population due to birth and death, heterogeneous beliefs, learning, and so on. As we stated, the consumption friction model is not inconsistent with long-term risk or rare disaster models, which have richer dynamics than the one-factor model we considered. Consideration of the richer aspects together with consumption frictions would be interesting.

It would be useful to investigate the welfare implications of the model, as in the habit model (Ljungqvist and Uhlig (2000, 2009)). It would be also interesting to study the monetary policy for household consumption and saving responses and their aggregate impact in an economy with consumption frictions.

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Appendix

A Preference with Adjustment Costs: Intertemporal Loss Aversion

Here we explain a utility function underlying (2). We first consider the utility function in discrete time and next derive its continuous-time limit. For an exogenously given consumption level c_{0-}, we consider the following utility function:

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} \hat{\delta}^t \left(u(c_t) - \alpha(\Delta u(c_t))^+ - \beta(\Delta u(c_t))^-\right)\right]. \tag{43}$$

Here $\mathbb{E}$ denotes expectation, $\hat{\delta}$ the subjective discount factor ($0 < \hat{\delta} < 1$), c_t consumption at time t, u is a strictly increasing concave real-valued function, called the *felicity* function, α, β are non-negative constants, $\Delta u(c_t) \equiv u(c_t) - u(c_{t-1})$ for $t > 0$ and $\Delta u(c_0) \equiv u(c_0) - u(c_{0-})$ and superscript $^+$ ($^-$) means the positive (negative) part of a real-valued function, i.e., $f^+ = \max(f, 0)$ ($f^- = \max(-f, 0)$). The first part of the utility function is the ordinary expected utility with felicity function u, the second part $\alpha(\Delta u(c_t))^+$ is the utility cost of adjusting consumption upward, and the third part $\beta(\Delta u(c_t))^-$ is the utility cost of adjusting consumption downward. Constant α (β) is a proportional cost of an upward (downward) consumption adjustment such that $\alpha \geq 0, \beta \geq 0$.[14]

We now briefly explain models with consumption adjustment costs. Grossman and Laroque (1990) introduced a model of a durable good, typically a house, in which its sale incurs monetary cost proportional to its value. Flavin and Nakagawa (2008) extend the model by adding a non-durable good and exogenously given prices of housing in different areas. Hindy and Huang (1992, 1993) propose a model of durable goods by introducing monetary adjustment cost proportional to the added value of durable goods. Hindy and Huang's model captures local substitutability, namely, consumptions at nearby time points are close substitutes. Chetty and Szeidl (2007, 2016) study consumption commitment by using a model built on that of Grossman and Laroque (1990). In Chetty and Szeidl's model adjustment of commitment goods is subject to monetary costs proportional to the sum of the purchase and sales values of the goods. Our model is close to Chetty and Szeidl's model; the difference between ours and the latter's lies in the fact that the costs are utility costs proportional to the changes in utility values in our model. Due to adjustment cost continuous adjustment of consumption is not optimal and adjustment occurs only sporadically when there is a large imbalance in all these models including ours. Thanks to the nature of costs, however, adjustment in our model is incremental and optimal consumption process is continuous, whereas consumption adjustment is lumpy and optimal consumption process exhibits discontinuity at adjustment times in other models. Furthermore, our model admits closed form solutions for an optimal consumption and investment problem (Choi et al. (2021)) and in a pure exchange economy as shown below. The other models except Hindy and Huang's admit only numerical solutions.[15]

We will next show that our preference model can be interpreted as a model of loss aversion toward consumption changes. Combining the permanent effect of a consumption adjustment and the adjustment cost, we can calculate the present value of utility gains and losses due to the adjustment. Let $\delta = 1 - \hat{\delta}$ be the subjective discount rate. The present value is equal to $(\frac{1}{\delta} - \alpha)(\Delta u(c_t))^+$ for an upward adjustment and the present value of utility loss (in absolute value) is equal to $(\frac{1}{\delta} + \beta)(\Delta u(c_t))^-$ for a downward adjustment. Hence the marginal utility of an increase in consumption is smaller than that of a decrease in consumption, implying that the agent exhibits loss aversion in consumption with the previous level of consumption being the reference point. There is evidence for loss aversion toward consumption changes in both micro and macro data (Hardie et al. (1993), Pasini (2009), Fisher and Montalto (2011), and Foellmi et al. (2019)).

Loss aversion in our model is *intertemporal* in the sense that a change in the current consumption has a persistent effect in the future. In accordance with the definition of loss aversion in the literature (e.g., Tversky and Kahneman (1991) and Benartzi and Thaler (1995)), we provide the following definition.

Definition 2. *We define the coefficient of ILA L to be the ratio of the marginal utility loss to the marginal utility gain, i.e.,*

$$L \equiv \frac{(\frac{1}{\delta} + \beta)|\Delta u(c)|}{(\frac{1}{\delta} - \alpha)|\Delta u(c)|} = \frac{1 + \delta\beta}{1 - \delta\alpha}. \tag{44}$$

Note that the agent would not increase consumption unless $\frac{1}{\delta} - \alpha > 0$, since the total utility gain would not be positive otherwise. This observation leads to the following assumption.

Assumption 2.

$$0 \leq \alpha < \frac{1}{\delta}.$$

[14]See Choi et al. (2021) for treatment of a general model with an arbitrary horizon and time-varying loss aversion. For example, if α and β are time-varying, the coefficient of ILA is time-varying according to Definition 2. In this paper, for simplicity we assume the coefficient is constant.

[15]We show in our companion paper Choi et al. (2020a) that our model can be regarded as an extension of Hindy and Huang's to allow resale of durable goods.

We will now show that the ILA uniquely determines the ordinal preference of the agent. Since $u(c_t) = u(c_{0-}) + \sum_{s=0}^{t} \Delta u(c_s)$ and $\Delta u(c_t) = (\Delta u(c_t))^+ - (\Delta u(c_t))^-$, we can rewrite the utility function (43) as

$$\begin{aligned} U &= \frac{u(c_{0-})}{\delta} + \mathbb{E}\left[\sum_{t=0}^{\infty} \hat{\delta}^t \left(\frac{\Delta u(c_t)}{\delta} - \alpha(\Delta u(c_t))^+ - \beta(\Delta u(c_t))^-\right)\right] \\ &= \frac{u(c_{0-})}{\delta} + \frac{1-\delta\alpha}{\delta}\mathbb{E}\left[\sum_{t=0}^{\infty} \hat{\delta}^t \left((\Delta u(c_t))^+ - L(\Delta u(c_t))^-\right)\right]. \end{aligned} \tag{45}$$

According to (45) the agent's utility consists of three parts: (i) the present value (PV) of the utility value of a constant level of consumption equal to c_{0-}, (ii) the PV of the increases in the utility value due to increases in consumption, and (iii) the absolute value of the decreases in the utility value due to decreases in consumption multiplied by the coefficient of ILA.

Notice that, given the same c_{0-}, the utility functions with two different pairs of parameters, (α_1, β_1) and (α_2, β_2), are affine transformations of the other if the two have the same coefficient L of ILA and they induce the same ordinal preference. Hence we regard the utility function (43) as representing an intertemporal preference with loss aversion in which the reference point for consumption at t is the previous level of consumption, c_{t-1} (c_{0-} if $t=0$). If the ILA is unity, i.e., $L=1$ (if and only if $\alpha = 0, \beta = 0$), then the agent is not loss-averse and the utility value is equal to the ordinary one. If the ILA is greater than 1, i.e, $L > 1$ (if and only if either $\alpha > 0$ or $\beta > 0$), then the agent exhibits loss aversion and the reduction in the utility value due to decreases in consumption have a higher weight equal to L.

We now consider a continuous-time limit of the utility function. Consider the following utility function

$$U = \mathbb{E}\left[\int_0^{\infty} e^{-\delta t}\left(u(c_t)dt - \alpha du_t^+ - \beta du_t^-\right)\right], \tag{46}$$

where the terms u_t^+ and u_t^- denote positive and negative variations of $u(c_t)$ over $[0-, t]$, which we will call positive and negative variation processes (here we denote a process $X = (X_t)_{t=0}^{\infty}$ simply by X_t).[16] The positive and negative variation processes can become infinite at a finite time. If the processes become infinite, then the corresponding utility costs are infinite. It is not optimal to adjust consumption so that the positive variation process becomes infinite in a finite time interval if $\alpha \neq 0$ and the negative variation process becomes infinite in a finite time interval if $\beta \neq 0$. The positive variation, negative variation and total variation are, however, all infinite if any one of them is infinite (Theorem 2.6, Wheeden (2015)).[17]

Similar to the discrete-time utility function in (45), we can rewrite the utility function (46) by using the ILA, as the following proposition shows. As in the discrete-time case, given c_{0-}, the ILA uniquely determines the agent's ordinal preference.

Proposition A.1. *The utility function (46) can be rewritten as follows:*

$$U = \frac{u(c_{0-})}{\delta} + \frac{1-\delta\alpha}{\delta}\mathbb{E}\left[\int_0^{\infty} e^{-\delta t}\left(du_t^+ - L du_t^-\right)\right]. \tag{47}$$

Proof. By integration by parts we can derive

$$\mathbb{E}\left[\int_0^T e^{-\delta t} u(c_t) dt\right] = \left[-\frac{e^{-\delta t}}{\delta} u(c_t)\right]_{t=0-}^{T} + \frac{1}{\delta}\mathbb{E}\left[\int_0^T e^{-\delta t} du(c_t)\right]. \tag{48}$$

Take $T \to \infty$. Since $\lim_{T\to\infty} \mathbb{E}\left[e^{-\delta T} u(c_T)\right] = 0$ and $u(c_t) = u(c_{0-}) + u^+ - u^-$, we have

$$\begin{aligned} U =& \frac{u(c_{0-})}{\delta} + \frac{1}{\delta}\mathbb{E}\left[\int_0^{\infty} e^{-\delta t} du(c_t) - \delta\alpha \int_0^{\infty} e^{-\delta t} du_t^+ - \delta\beta \int_0^{\infty} e^{-\delta t} du_t^-\right] \\ =& \frac{u(c_{0-})}{\delta} + \frac{1}{\delta}\mathbb{E}\left[\int_0^{\infty} e^{-\delta t}\left((1-\delta\alpha)du_t^+ - (1+\delta\beta)du_t^-\right)\right] \\ =& \frac{u(c_{0-})}{\delta} + \frac{1-\delta\alpha}{\delta}\mathbb{E}\left[\int_0^{\infty} e^{-\delta t} du_t^+ - L du_t^-\right]. \end{aligned} \tag{49}$$

□

[16] The positive (negative) variation over $[T_1, T_2]$ of a process X_t is defined as $\sup_{[t_0, t_1, \ldots t_N]} \sum (X_{t_{i+1}} - X_{t_i})^+$ ($\sup_{[t_0, t_1, \ldots t_N]} \sum (X_{t_{i+1}} - X_{t_i})^-$) where $[t_0, t_1, \ldots t_N]$ is an arbitrary partition of $[T_1, T_2]$. The total variation is defined as the sum of the positive and negative variations.

[17] Accordingly, if the consumption process c_t has an infinite total variation over a finite time interval $[0-, t]$, then the process $u(c_t)$ also has an infinite total variation and infinite positive and negative variations, and the utility costs are infinite if $(\alpha, \beta) \neq (0, 0)$; thus, we can preclude any consumption policy which results in a process c_t with an infinite total variation over a finite time interval as sub-optimal.

Before we close this section, here is a summarizing remark on the preference. The utility function displays loss aversion toward consumption changes in which the reference point is the most recent consumption level. It is a natural extension of the canonical time-separable utility function, exhibiting a minimal departure from the latter; it has only one extra parameter, *the coefficient of intertemporal loss aversion*, besides the subjective discount rate and the risk aversion coefficient.[18]

B Interpretation as a single-good heterogeneous agent economy

We can reinterpret our basic model as that of a single-good heterogeneous agent economy.

There are two classes of identical agents, and they are represented by agent L (a loss-averse agent) and agent N (a non-loss-averse agent). Agent L has a utility function (46) with $(\alpha, \beta) \neq 0$ and agent N has a utility function (46) with $(\alpha, \beta) = 0$.

There is a financial market in which claims to the future endowment flow and instantaneous investment vehicles are traded. The claim to the full endowment flow is called *equity* or *stock*. The total number of shares of the stock is 1. Each agent is endowed with shares of the stock: we denote the number of shares endowed to agent i (i =L, N) by $N^S_{i,0-}$. The instantaneous investment vehicles are called (instantaneous) *bonds*, and if one invests one unit of consumption at time t in the bonds one gets $1 + dR_{f,t}$ at time $t + dt$. Financial claims are traded with no friction, i.e. there exist neither transaction cost nor short-selling restriction on bonds and stocks. The bonds are in zero net supply and each agent's initial endowment of bonds is equal to 0. Thus, denoting the number of shares and bonds owned by agent L and agent N at time $t \geq 0$ by $N^S_{L,t}, N^S_{N,t}$, and $N^B_{L,t}, N^B_{N,t}$, respectively, we have the following market clearing conditions:

$$N^S_{L,t} + N^S_{N,t} = 1, \qquad N^B_{L,t} + N^B_{N,t} = 0, \quad t \geq 0. \tag{50}$$

Since there exists only one source of risk, with no trading friction, the financial market is dynamically complete, i.e., all contingent claims to the economy's future endowment can be replicated by trading financial assets (Duffie and Huang, 1985).

We denote the price of a share of the stock by S_t and normalize the price of a bond to be 1. Agent i's wealth $W_{i,t}$ at time t is given by $W_{i,t} = N^S_{i,t} S_t + N^B_{i,t}$. Over the time interval $[t, t + dt)$ the agent consumes a fraction of wealth, while receiving dividends distributed proportional to share holdings. Accordingly, the wealth change over the time interval satisfies the following dynamics:

$$dW_{i,t} = N^S_{i,t}(y_t dt + dS_t) + N^B_{i,t} dR_{f,t} - c_{i,t} dt,$$

where $c_{i,t}$ denotes her consumption at t and the equilibrium dynamics of the stock price S_t and the bond return $dR_{f,t}$ are described in Section 4. We require that the agent is always solvent, keeping non-negative wealth, i.e.,

$$W_{i,t} \geq 0 \qquad \text{for all } t \geq 0, \tag{51}$$

Finally we define the competitive equilibrium of the economy, following the definition by Radner (1972).

Definition 3 (Market equilibrium).
A competitive equilibrium of the economy is the pair of price process $(S_t, R_{f,t})$ and consumption-portfolio strategies $(c_{i,t}, N^S_{i,t}, N^B_{i,t})$, i =L,N, such that

(i) the strategy maximizes each agent's utility function subject to wealth evolution equation (51), where $W_{i,0-} = N^S_{i,0-} S_0$, and

(ii) the market clearing condition for the stock and bonds:

$$N^S_{L,t} + N^S_{N,t} = 1, \quad N^B_{L,t} + N^B_{N,t} = 0 \qquad \text{for all } t \geq 0. \tag{52}$$

We can obtain the competitive equilibrium by using a standard approach. We first solve the planner's problem, then consider the Arrow-Debreu economy, and finally obtain the competitive equilibrium. The competitive equilibrium is essentially the same as that of the representative agent economy with heterogeneous goods.

C Solution to the Planner's Problem

In this appendix, we will obtain a solution to the planner's problem by solving its associated HJB equation.

According to Choi et al. (2021), the social planner's problem can be rewritten as

$$V(y, c) = \sup_{c_F, c_A} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{w\left(u(c_{F,t})dt - \alpha u'(c_{F,t})dc^+_{F,t} - \beta u'(c_{F,t})dc^-_{F,t}\right) + (1-w)u(c_{A,t})dt\right\}\right] \tag{53}$$

subject to $c_{F,t} + c_{A,t} \leq y_t$ for all $t \geq 0$, where $c^+_{F,t}$ and $c^-_{F,t}$ denote positive and negative variations of $c_{F,t}$ over $[0-, t]$ and thus $c_{F,t} = c_{0-} + c^+_{F,t} - c^-_{F,t}$.

[18] Our companion papers provide the microfoundation of the intertemporal preference (Choi et al., 2021) and its extension and implications for aggregate consumption and asset management (Choi et al., 2020a).

Similar to a standard approach for a *singular control problem* (see e.g., Davis and Norman (1990), Fleming and Soner (2006)), we will show that $V(y,c)$ will satisfy the following HJB-equation:

$$\max\left\{\mathcal{L}V(y,c)+wu(c)+(1-w)u(y-c),\ V_c(y,c)-w\alpha u'(c),\ -V_c(y,c)-w\beta u'(c)\right\}=0, \tag{54}$$

where the differential operator $\mathcal{L}$ is given by $\mathcal{L}=\dfrac{\sigma^2}{2}y^2\dfrac{d^2}{dy^2}+\mu y\dfrac{d}{dy}-\delta$.

According to Section 4.1, the F-consumption can be described by three regions, the inaction region (**NR**), the increasing region (**IR**), and the decreasing region (**DR**) of the state space $\mathcal{D}=\{(y,c)\mid 0<y<\infty,\ \ 0<c<y\}$.

$$\begin{aligned}\mathbf{IR}&\equiv\{(y,c)\in\mathcal{D}\mid V_c(y,c)=w\alpha u'(c)\},\\ \mathbf{NR}&\equiv\{(y,c)\in\mathcal{D}\mid -w\beta u'(c)<V_c(y,c)<w\alpha u'(c)\},\\ \mathbf{DR}&\equiv\{(y,c)\in\mathcal{D}\mid V_c(y,c)=-w\beta u'(c)\}.\end{aligned} \tag{55}$$

Thanks to the isoesatic utility function (5), the social welfare function is homogeneous in (y,c) and thus we can reduce the dimension of the problem by introducing the variable

$$H(z_t)=\frac{V_c(y_t,c_{t-})}{u'(c_{t-})}\quad\text{and}\quad z_t=\frac{y_t}{c_{t-}}. \tag{56}$$

We will call the variable z_t the aggregate endowment to consumption ratio. In the inaction region **NR**, $H(z)$ satisfies the following ordinary differential equation(ODE):

$$\frac{\sigma^2z^2}{2}z^2\frac{dH^2}{dz^2}+\mu z\frac{dH}{dz}+h(z)-\delta H=0,\ \text{ where }\ z\equiv\frac{y}{c}, \tag{57}$$

where $h(z)=w-(1-w)(z-1)^{-\gamma}$. Let m_1 and m_2 be the positive and negative roots of the following quadratic equation:

$$\frac{\sigma^2}{2}m^2+\left(\mu-\frac{\sigma^2}{2}\right)m-\delta=0. \tag{58}$$

Then, the solution to ODE (57) takes the following form:

$$H(z)=D_1\left(\frac{z}{z_2}\right)^{m_1}+D_2\left(\frac{z}{z_2}\right)^{m_2}+\eta(z), \tag{59}$$

where D_1,D_2 are constants to be determined later and $\eta(z)$ is given by

$$\eta(z)=\frac{2}{\sigma^2(m_1-m_2)}\left[z^{m_2}\int_{z_2}^{z}v^{-m_2-1}h(v)dv-z^{m_1}\int_{z_2}^{z}v^{-m_1-1}h(v)dv\right]. \tag{60}$$

There exist free boundaries $z_1>0$ and $z_2>0$ such that the boundary conditions become

$$H(z_1)=w\alpha,\ H'(z_1)=0,\ H(z_2)=-w\beta,\ H'(z_2)=0,\qquad\text{and} \tag{61}$$

$$\begin{cases}H(z)=w\alpha, & \text{if }\ z\geq z_1,\\ H(z)=-w\beta, & \text{if }\ z\leq z_2.\end{cases} \tag{62}$$

The boundary conditions (61) imply that D_1 and D_2 are given by

$$D_1=\frac{w\beta m_2}{m_1-m_2},\quad D_2=-\frac{w\beta m_1}{m_1-m_2}, \tag{63}$$

and boundaries z_1,z_2 are uniquely determined as solutions to the coupled algebraic equations:

$$\begin{aligned}f(v_1,v_2)&=\int_{v_2}^{v_1}v^{-m_1-1}(w(1-\alpha\delta)-(1-w)(v-1)^{-\gamma})dv-\frac{\sigma^2}{2}m_2w(\alpha+\beta)\frac{1}{v_2^{m_1}}=0,\\ g(v_1,v_2)&=\int_{v_2}^{v_1}v^{-m_2-1}(w(1-\alpha\delta)-(1-w)(v-1)^{-\gamma})dv-\frac{\sigma^2}{2}m_1w(\alpha+\beta)\frac{1}{v_2^{m_2}}=0.\end{aligned} \tag{64}$$

In the next proposition, we will show that solution (z_2,z_1) to the above system of algebraic equations is unique.

Proposition C.1. *The solution* (z_1, z_2) *to the system of algebraic equation* (64) *is unique. Moreover,*

$$1 < z_2 < \underline{z} := 1 + \left(\frac{w(1+\beta\delta)}{(1-w)}\right)^{-\frac{1}{\gamma}}, \quad \bar{z} := 1 + \left(\frac{w(1-\alpha\delta)}{(1-w)}\right)^{-\frac{1}{\gamma}} < z_1 < \infty.$$

Proof. The proof is given in Section S.1 of the Supplemental Material. □

By using the closed-form solution $H(z)$ and the relation (56), we will conjecture the value function $V(y, c)$ and verify it. The scale independence (or homotheticity) for the CRRA implies that there exists a function $v(\cdot)$ such that

$$V(y,c) = \begin{cases} c^{1-\gamma} v(y/c), & \text{if } 0 < \gamma \text{ and } \gamma \neq 1, \\ \dfrac{\log c}{\delta} + v(y/c), & \text{if } \gamma = 1. \end{cases} \tag{65}$$

Since $v(z)$ satisfies

$$Lv(z) + wu(1) + (1-w)u(z) = 0, \quad \text{for } (y,c) \in \mathbf{NR},$$

the general solution $v(z)$ can be written as the sum of a general solution to the homogeneous equation and a particular solution:

$$v(z) = A_1 z^{m_1} + A_2 z^{m_2} + \zeta(z), \tag{66}$$

where $\zeta(z)$ is given by

$$\zeta(z) = \frac{2}{\sigma^2(m_1 - m_2)} \left[z^{m_2} \int_{z_2}^{z} v^{-m_2 - 1} q(v) dv - z^{m_1} \int_{z_2}^{z} v^{-m_1 - 1} q(v) dv \right] \tag{67}$$

with $q(z) = wu(1) + (1-w)u(z-1)$. From the relation (65), we deduce that

$$V_c(y,c) = \begin{cases} c^{-\gamma}((1-\gamma)v(z) - zv'(z), & \text{if } 0 < \gamma \text{ and } \gamma \neq 1, \\ c^{-1}(1/\delta - zv'(z)), & \text{if } \gamma = 1. \end{cases} \tag{68}$$

It follows from (56), (59) and (66) that for $(y,c) \in \mathbf{NR}$,

$$v(z) = \frac{D_1}{(1-\gamma-m_1)} z^{m_1} + \frac{D_2}{(1-\gamma-m_2)} z^{m_2} + q(z). \tag{69}$$

Since

$$\begin{cases} V_c(y,c) = w\alpha u'(c) & \text{for } (y,c) \in \mathbf{IR}, \\ V_c(y,c) = -w\beta u'(c) & \text{for } (y,c) \in \mathbf{DR}, \end{cases} \tag{70}$$

we can construct the value function $V(y, c)$ by

(i) for $z_2 < y/c < z_1$,

$$V(y,c) = \begin{cases} c^{1-\gamma} \left(\frac{D_1}{(1-\gamma-m_1)} \left(\frac{y}{c}\right)^{m_1} + \frac{D_2}{(1-\gamma-m_2)} \left(\frac{y}{c}\right)^{m_2} + \zeta\left(\frac{y}{c}\right) \right), & \text{if } \gamma \neq 1, \\ \frac{\log c}{\delta} - \frac{D_1}{m_1} \left(\frac{y}{c}\right)^{m_1} - \frac{D_2}{m_2} \left(\frac{y}{c}\right)^{m_2} + \zeta\left(\frac{y}{c}\right), & \text{if } \gamma = 1. \end{cases} \tag{71}$$

(ii) for $y/c \geq z_1$,

$$V(y,c) = V\left(y, \frac{y}{z_1}\right) + w\alpha \left(u(c) - u\left(\frac{y}{z_1}\right) \right). \tag{72}$$

(iii) for $y/c \leq z_2$,

$$V(y,c) = V\left(y, \frac{y}{z_2}\right) - w\beta \left(u(c) - u\left(\frac{y}{z_2}\right) \right). \tag{73}$$

We now conjecture the explicit form of the value function $V(y, c)$. In fact, the following proposition can be established.

Proposition C.2. *The value function $V(y,c)$ given in* (71), (72) *and* (73) *satisfies the HJB-equation* (54) *and the regions* **IR**, **NR**, *and* **DR** *are rewritten by*

$$\mathbf{IR} = \{(y,c) \in \mathcal{D} \mid 0 < c/y \leq \underline{c}\}, \quad \mathbf{NR} = \{(y,c) \in \mathcal{D} \mid \underline{c} < c/y < \bar{c}\}, \quad \mathbf{DR} = \{(y,c) \in \mathcal{D} \mid \bar{c} \leq c/y < \infty\}, \tag{74}$$

where $\bar{c} = \dfrac{1}{z_2}$ *and* $\underline{c} = \dfrac{1}{z_1}$.

Proof. The proof is given in Section S.2 of the Supplemental Material. □

Finally, in the next theorem, we verify that the conjectured form of $V(y,c)$ given in (71), (72) and (73) is a solution to the planner's problem defined in Problem 2.

Theorem C.1 (Verification theorem)**.**

1. *Suppose that the HJB equation* (54) *has a twice continuously differentiable solution* $V(y,c): \mathcal{D} \to \mathbb{R}$ *satisfying the following conditions:*

   (1) For any F-consumption strategy $(c_{F,t})_{t=0}^{\infty}$*, the process defined by*

$$\int_0^t e^{-\delta s} \sigma y_s V_y(y_s, c_{F,s}) dB_s, \quad t \geq 0,$$

   is a martingale.

   (2) For any F-consumption strategy $(c_{F,t})_{t=0}^{\infty}$*,*

$$\liminf_{t\to\infty} \mathbb{E}\left[e^{-\delta t} V(y_t, c_{F,t})\right] \geq 0.$$

   Then, for initial condition $(y,c) \in \mathcal{D}$ *and any F-consumption strategy* $(c_{F,t})_{t=0}^{\infty}$*,*

$$V(y,c) \geq \sup_{c_F, c_A} \mathbb{E}\left[\int_0^\infty e^{-\delta t}\left\{w\left(u(c_{F,t})dt - w\alpha u(c_{L,t})dc_{F,t}^{+} - w\beta u(c_{F,t})dc_{F,t}^{-}\right) + (1-w)u(c_{A,t})dt\right\}\right].$$

2. *Given any initial* $(y,c) \in \mathcal{D}$*, suppose that there exists a continuous F-consumption strategy* $(c_{F,t}^*)_{t=0}^{\infty}$ *such that*

$$\begin{cases} (y_t, c_{F,t}^*) \in \{(y,c) \in \mathcal{D} \mid \mathcal{L}V(y,c) + wu(c) + (1-w)u(y-c)\}, & \mathbb{P}-a.s., \\ \displaystyle\int_0^t e^{-\delta s}\left(V_c(y_t, c_{F,s}^*) - w\alpha u'(c_{F,s}^*)\right) dc_{F,s}^{*,+} = 0, \quad \text{for all } t \geq 0, & \mathbb{P}-a.s., \\ \displaystyle\int_0^t e^{-\delta s}\left(-V_c(y_t, c_{F,s}^*) - w\beta u'(c_{F,s}^*)\right) dc_{F,s}^{*,-} = 0, \quad \text{for all } t \geq 0, & \mathbb{P}-a.s.. \end{cases} \tag{75}$$

   Then, $(c_{F,t}^*)_{t=0}^{\infty}$ *and the* $V(y,c)$ *given in* (71), (72), *and* (73) *are the optimal F-consumption strategy and the value function for the social planner's problem, respectively.*

Proof. The proof is given in Section S.3 of the Supplemental Material. □

As similar to Choi et al. (2021), we can express the optimal F-consumption strategy $(c_{F,t}^*)_{t=0}^{\infty}$ as

$$c_{F,t}^{*,+} = \max\left\{0, -c_{0-} + \sup_{s\in[0,t)}\left(c_{F,s}^{*,-} + \underline{c}y_s\right)\right\}, \quad c_{F,t}^{*,-} = \max\left\{0, \ c_{0-} + \sup_{s\in[0,t)}\left(c_{F,s}^{*,+} - \bar{c}y_s\right)\right\}, \tag{76}$$

where $c_F^{*,+}$ and $c_F^{*,-}$ are positive and negative variation processes of c_F^*, i.e.,

$$c_{F,t}^* = c + c_{F,t}^{*,+} - c_{F,t}^{*,-}.$$

It is not difficult to show that the optimal F-consumption strategy $(c_{F,t}^*)_{t=0}^{\infty}$ given in (76) satisfies the assumption in Theorem C.1.

D Proof of Proposition 2.2

(a) Since z_1 and z_2 satisfy the following couple-algebraic equations

$$\begin{aligned} f(z_1,z_2) &= \int_{z_2}^{z_1} v^{-m_1-1}(w(1-\alpha\delta)-(1-w)(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_2 w(\alpha+\beta)\frac{1}{z_2^{m_1}} = 0,\\ g(z_1,z_2) &= \int_{z_2}^{z_1} v^{-m_2-1}(w(1-\alpha\delta)-(1-w)(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_1 w(\alpha+\beta)\frac{1}{z_2^{m_2}} = 0, \end{aligned} \tag{77}$$

we obtain

$$\begin{aligned} z_1^{-m_1-1}(w(1-\alpha\delta)-(1-w)(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial\beta} - z_2^{-m_1-1}(w(1+\beta\delta)-(1-w)(z_2-1)^{-\gamma})\frac{\partial z_2}{\partial\beta} &= \frac{\sigma^2}{2}w\alpha m_2\frac{1}{z_2^{m_1}},\\ z_1^{-m_2-1}(w(1-\alpha\delta)-(1-w)(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial\beta} - z_2^{-m_2-1}(w(1+\beta\delta)-(1-w)(z_2-1)^{-\gamma})\frac{\partial z_2}{\partial\beta} &= \frac{\sigma^2}{2}w\alpha m_1\frac{1}{z_2^{m_2}}. \end{aligned} \tag{78}$$

This leads to

$$z_1^{-m_1-1}z_2^{-m_2-1}\left(1-\left(\frac{z_1}{z_2}\right)^{m_1-m_2}\right)(w(1-\alpha\delta)-(1-w)(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial\beta} = z_2^{-m_1-m_2-1}\frac{\sigma^2}{2}w\alpha(m_2-m_1). \tag{79}$$

It follows from $1<z_2<\underline{z}<\bar{z}<z_1$ that

$$\frac{\partial z_1}{\partial\beta}>0 \text{ and } \frac{\partial z_2}{\partial\beta}<0.$$

Note that we can rewrite the equations (77) as follows:

$$\begin{aligned} &\int_{z_2}^{z_1} v^{-m_1-1}(w(1+\beta\delta)-(1-w)(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_2 w(\alpha+\beta)\frac{1}{z_1^{m_1}} = 0,\\ &\int_{z_2}^{z_1} v^{-m_2-1}(w(1+\beta\delta)-(1-w)(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_1 w(\alpha+\beta)\frac{1}{z_1^{m_2}} = 0. \end{aligned} \tag{80}$$

Similarly, we can easily derive that

$$\frac{\partial z_1}{\partial\alpha}>0 \text{ and } \frac{\partial z_2}{\partial\alpha}<0.$$

Since $\bar{c}=1/z_2, \underline{c}=1/z_1$ and $L=(1+\beta\delta)/(1-\delta\alpha)$, we conclude that $\bar{c}$ increases with L and $\underline{c}$ decreases with L.

(b) Since

$$\begin{aligned} f(z_1,z_2) &= \int_{z_2}^{z_1} v^{-m_1-1}((1-\alpha\delta)-\frac{(1-w)}{w}(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_2(\alpha+\beta)\frac{1}{z_2^{m_1}} = 0,\\ g(z_1,z_2) &= \int_{z_2}^{z_1} v^{-m_2-1}((1-\alpha\delta)-\frac{(1-w)}{w}(v-1)^{-\gamma})dv - \frac{\sigma^2}{2}m_1(\alpha+\beta)\frac{1}{z_2^{m_2}} = 0, \end{aligned} \tag{81}$$

we have

$$\begin{aligned} z_1^{-m_1-1}((1-\alpha\delta)-\frac{(1-w)}{w}(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial w} - z_2^{-m_1-1}((1+\beta\delta)-\frac{(1-w)}{w}(z_2-1)^{-\gamma})\frac{\partial z_2}{\partial w} &= -\frac{\sigma^2}{2w^2}\int_{z_2}^{z_1} v^{-m_1-1}(v-1)^{-\gamma},\\ z_1^{-m_2-1}((1-\alpha\delta)-\frac{(1-w)}{w}(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial w} - z_2^{-m_2-1}((1+\beta\delta)-\frac{(1-w)}{w}(z_2-1)^{-\gamma})\frac{\partial z_2}{\partial w} &= -\frac{\sigma^2}{2w^2}\int_{z_2}^{z_2} v^{-m_2-1}(v-1)^{-\gamma}. \end{aligned}$$

This implies that

$$\begin{aligned} &z_1^{-m_1-1}z_2^{-m_2-1}\left(1-\left(\frac{z_1}{z_2}\right)^{m_1-m_2}\right)((1-\alpha\delta)-\frac{(1-w)}{w}(z_1-1)^{-\gamma})\frac{\partial z_1}{\partial w}\\ &= -\frac{\sigma^2}{2w^2}\int_{z_2}^{z_1} v^{-m_1-1}z_2^{-m_2-1}\left(1-\left(\frac{v}{z_2}\right)^{m_1-m2}\right)(v-1)^{-\gamma}dv \end{aligned} \tag{82}$$

and

$$
\begin{aligned}
& z_1^{-m_1-1} z_2^{-m_2-1}\left(1-\left(\frac{z_1}{z_2}\right)^{m_1-m_2}\right)((1+\beta\delta)-\frac{(1-w)}{w}(z_2-1)^{-\gamma})\frac{\partial z_1}{\partial w} \\
= & -\frac{\sigma^2}{2w^2}\int_{z_2}^{z_1} v^{-m_1-1} z_1^{-m_2-1}\left(1-\left(\frac{v}{z_1}\right)^{m_1-m_2}\right)(v-1)^{-\gamma}dv.
\end{aligned}
\tag{83}
$$

It follows that

$$
\frac{\partial z_1}{\partial w} < 0 \quad \text{and} \quad \frac{\partial z_2}{\partial w} < 0.
$$

Thus, we have $\bar{c}$ and $\underline{c}$ increase with w.

E Derivation of a stationary density for the F-share

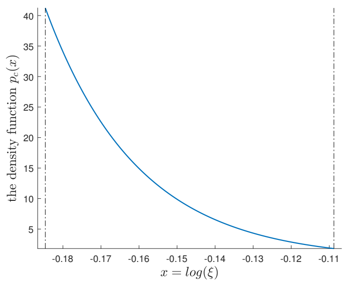

Figure 2: Stationary distributions for $\log c_t^*/y_t$. Parameter values are as follows: $\mu = 0.023, \sigma = 0.033, \delta = 0.01, \gamma = 1, L = 1.005, w = 0.85, y_0 = 1$, and $c_{0-} = 0.8$. With this parameter set, $\underline{c} = 0.7966$ and $\bar{c} = 0.9183$.

By Proposition 5.5 in Harrison (1985) or Proposition 10.8 in Stokey (2009) we derive the stationary distribution $p_c(x)$ for the $\log \mathcal{F}_t = \log c_t^*/y_t$; the density function for the stationary distribution is given by

$$
p_c(x) = \frac{\kappa e^{\kappa x}}{\bar{c}^{\kappa} - \underline{c}^{\kappa}}, \quad x \in [\log \underline{c}, \log \bar{c}], \tag{84}
$$

where $\kappa \equiv 1 - \frac{2\mu}{\sigma^2}$. Figure 2 plots the stationary distributions of the consumption share. Then, we can compute all the asset pricing moments including conditional expectations by using (84).

F Proof that the EIS of consumption A is equal to $1/\gamma$

Itô's lemma implies that

$$
\frac{d\mathcal{H}_t}{\mathcal{H}_t} = -\delta dt + \frac{c_{A,t}u''(c_{A,t})}{u'(c_{A,t})}\frac{dc_{A,t}}{c_{A,t}} + \frac{1}{2}\frac{c_{A,t}^2 u'''(c_{A,t})}{u'(c_{A,t})}\frac{< dc_{A,t}, dc_{A,t} >}{c_{A,t}^2}, \text{[19]} \tag{85}
$$

where $< dc_{A,t}, dc_{A,t}, >$ denotes the quadratic variation of $dc_{A,t}$. Thus,

$$
\mathbb{E}_t\left[\frac{dc_{A,t}}{c_{A,t}}\right] = \delta dt - \frac{1}{2}\frac{c_{A,t}^2 u'''(c_{A,t})}{u'(c_{A,t})}\frac{< dc_{A,t}, dc_{A,t} >}{c_{A,t}^2} + \frac{1}{\gamma}\mathbb{E}_t\left[\frac{d\mathcal{H}_t}{\mathcal{H}_t}\right]. \tag{86}
$$

This shows that the EIS of agent N is equal to $1/\gamma$.

[19] The equation is the same as equation (37), Ch. 1 Cochrane (2008).

G Derivation of prices of the stock and consol bonds in the basic model

We will derive the price P_t of a consol bond and S_t of the equity.

We know

$$P_t = \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)}\frac{\mathcal{H}_s}{\mathcal{H}_t}ds|\mathcal{F}_t\right] = (c_{A,t})^\gamma \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)}(c_{A,s})^{-\gamma}ds\right] = (y_t - c^*_{F,t})^\gamma V_B(y_t, c^*_{F,t}), \tag{87}$$

where

$$V_B(y,c) \equiv \mathbb{E}\left[\int_t^\infty e^{-\delta(s-t)}(y_s - c^*_{F,s})^{-\gamma}ds \mid y_t = y,\ c^*_{F,t} = c\right].$$

Then, $V_B(y,c)$ satisfies the following PDE:

$$\begin{cases} \mathcal{L}V_B(y,c) + (y-c)^{-\gamma} = 0 \quad \text{for } cz_2 < y < cz_1. \\ \partial_c V_B(y, y/z_1) = \partial_c V_B(y, y/z_2) = 0, \end{cases} \tag{88}$$

Let us consider the following substitution:

$$V_B(y,c) = c^{-\gamma}H_B(y/c) \ \text{ with } \ z = y/c.$$

It follows from (88) that $H_B(z)$ satisfies the ODE:

$$\begin{cases} \mathcal{L}H_B(z) + (z-1)^{-\gamma} = 0 \text{ for } z_2 < z < z_1, \\ -\gamma H_B(z_1) - z_1 H'_B(z_1) = -\gamma H_B(z_2) - z_2 H'_B(z_2) = 0. \end{cases} \tag{89}$$

A solution to ODE (89) can be written as the sum of a general solution to the homogeneous equation and a particular solution:

$$H_B(z) = G_1 z^{m_1} + G_2 z^{m_2} + \phi_B(z), \tag{90}$$

where $\phi_B(z)$ is given by

$$\phi_B(z) = \frac{2}{\sigma^2(m_1 - m_2)}\left[z^{m_2}\int_{z_2}^z v^{-m_2-1}(v-1)^{-\gamma}dv - z^{m_1}\int_{z_2}^z v^{-m_1-1}(v-1)^{-\gamma}dv\right]. \tag{91}$$

From the boundary condition of $H_B(z)$ at $z = z_1$ and z_2, we derive the coefficients D_1 and $D_{,2\lambda}$:

$$G_1 = \frac{(-\gamma\phi_B(z_1) - z_1\phi'_B(z_1))z_2^{m_2}}{(-\gamma - m_1)(z_1^{m_2}z_2^{m_1} - z_1^{m_1}z_2^{m_2})}, \quad G_2 = \frac{-(\gamma\phi_B(z_1) + z_1\phi'_B(z_1))z_2^{m_1}}{(-\gamma - m_2)(z_1^{m_2}z_2^{m_1} - z_1^{m_1}z_2^{m_2})}. \tag{92}$$

Thus, the price P_t of a consol bond is given by

$$P_t = \mathcal{F}_t^{-\gamma}(1-\mathcal{F}_t)^\gamma\left(G_1\mathcal{F}_t^{-m_1} + G_2\mathcal{F}_t^{-m_2} + \phi_B(\mathcal{F}_t^{-1})\right). \tag{93}$$

We now derive the price S_t of the equity. We know

$$\mathcal{H}_t S_t = \mathbb{E}_t\left[\int_t^\infty \mathcal{H}_s y_s ds|\mathcal{F}_t\right]. \tag{94}$$

Since $\mathcal{H}_t = e^{-\delta t}(c^*_{A,t})^{-\gamma}$ in (17), we deduce that

$$S_t = \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)}\frac{\mathcal{H}_s}{\mathcal{H}_t}y_s ds|\mathcal{F}_t\right] = (c^*_{A,t})^\gamma \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)}(c^*_{A,s})^{-\gamma}y_s ds\right] = (y_t - c^*_{F,t})^\gamma V_S(y,c), \tag{95}$$

where

$$V_S(y,c) \equiv \mathbb{E}\left[\int_t^\infty e^{-\delta(s-t)}y_s(y_s - c^*_{F,s})^{-\gamma}ds \mid y_t = y,\ c^*_{F,t} = c\right].$$

Then, $V_S(y,c)$ satisfies the following PDE:

$$\begin{cases} \mathcal{L}V_S(y,c) + y(y-c)^{-\gamma} = 0 \text{ for } cz_2 < y < cz_1, \\ \partial_c V_S(y, y/z_1) = \partial_c V_S(y, y/z_2) = 0. \end{cases} \tag{96}$$

Let us consider the following substitution:

$$V_S(y,c) = c^{1-\gamma} H_S(y/c) \ \text{ with } \ z = y/c.$$

It follows from (96) that $H_S(z)$ satisfies the following ordinary differential equation(ODE):

$$\begin{cases} \mathcal{L}H_S(z) + z(z-1)^{-\gamma} = 0 \text{ for } z_2 < z < z_1, \\ (1-\gamma)H_S(z_1) - z_1(H_S)'(z_1) = (1-\gamma)H_S(z_2) - z_2(H_S)'(z_2) = 0, \end{cases} \tag{97}$$

A solution to ODE (97) can be written as the sum of a general solution to the homogeneous equation and a particular solution:

$$H_S(z) = E_1 z^{m_1} + E_2 z^{m_2} + \phi_S(z), \tag{98}$$

where $\phi_S(z)$ is given by

$$\phi_S(z) = \frac{2}{\sigma^2(m_1 - m_2)} \left[z^{m_2} \int_{z_2}^{z} v^{-m_2}(v-1)^{-\gamma} dv - z^{m_1} \int_{z_2}^{z} v^{-m_1}(v-1)^{-\gamma} dv \right]. \tag{99}$$

From the boundary condition of $H_S(z)$ at $z = z_1, \ z_2$, we deduce that the coefficients F_1 and F_2 are given by

$$E_1 = \frac{((1-\gamma)\phi_S(z_1) - z_1(\phi_S)'(z_1))z_2^{m_2}}{(1-\gamma-m_1)(z_1^{m_1} z_2^{m_2} - z_1^{m_2} z_2^{m_1})}, \quad E_2 = \frac{((1-\gamma)\phi_S(z_1) - z_1(\phi_S)'(z_1))z_2^{m_1}}{(1-\gamma-m_2)(z_1^{m_1} z_2^{m_2} - z_1^{m_2} z_2^{m_1})}. \tag{100}$$

Thus, the price S_t of the equity is given by

$$S_t = (1-\mathcal{F}_t)^{\gamma} \mathcal{F}_t^{1-\gamma} \left(E_1 \mathcal{F}_t^{-m_1} + E_2 \mathcal{F}_t^{-m_2} + \phi_S\left({\mathcal{F}_t}^{-1}\right) \right) y_t. \tag{101}$$

By the generalized Itô's lemma and (101) we can derive

$$dS_t/S_t = (\mu_{s,t}dt - dG_t) + \sigma_{s,t} dB_t, \tag{102}$$

where we have used the fact $\left\{(1-\gamma)H_S(\mathcal{F}_t^{-1}) - \mathcal{F}_t^{-1} H_S(\mathcal{F}_t^{-1})\right\} dc_t^* = 0$ and $\mu_{s,t}, \sigma_{s,t}$ are given by

$$\mu_{s,t} = r_t + \sigma_{s,t}\theta_t - \frac{y_t}{S_t}, \quad \sigma_{s,t} = \sigma \left(\frac{\gamma}{1-\mathcal{F}_t} + \frac{(m_1 E_1 \mathcal{F}_t^{-m_1} + m_2 E_2 \mathcal{F}_t^{-m_2} + \mathcal{F}_t^{-1}(\phi_S)'(\mathcal{F}_t^{-1}))}{(E_1 \mathcal{F}_t^{-m_1} + E_2 \mathcal{F}_t^{-m_2} + (\phi_S)(\mathcal{F}_t^{-1}))} \right). \tag{103}$$

H A Model of Heterogeneous Adjustments

We consider an extension of the basic model. There exists m different agents. Each agent can represent a group of identical agents as usually in an equilibrium model. Agent i has adjustment costs parameterized by $(\alpha_i, \beta_i), \ i = 1, \dots, m$. Previous levels of their F-consumption are given by $c_{0-} = (c_{i,0-})_{i=1}^m$ at time t. Agents experience preference shocks which changes their adjustment costs $(\alpha_i, \beta_i), \ i = 1, \dots, m$. Shocks are driven by independent Poisson events.

The planner optimizes the following social welfare function.

$$\mathcal{U} = \sum_{i=1}^{m} w_i \left\{ w_{F,i} U_{F,i} + (1 - w_{F,i}) U_{A,i} \right\}, \tag{104}$$

where U_F is the expected utility of agent i's F-consumption, $U_{A,i}$ is the expected utility of his(her) A-consumption, w_i is the weight of agent i in the social utility function, $w_{F,i}$ is the utility weight of i's F-consumption.

The aggregate output y_t follows the geometric Brownian motion (1) as in the basic model. Let us denote agent i's F-consumption by $c_i = (c_{i,t})_{t \geq 0}$.

The value function V of the planner is a function of $y_t, c_{t-} = (c_{i,t-})_{i=1}^m, \Delta_t = ((\alpha_{i,t}, \beta_{i,t})_{i=1}^m)$.

The optimality conditions for adjusting agent i's F-consumption are

$$\frac{\partial V}{\partial c_{i,t-}} V(y_t, c_{i,t-}) = w_i w_{F,i} \alpha_{i,t} u'(c_{i,t-}) \tag{105}$$

when the planner increases agent i's F-consumption, and

$$\frac{\partial V}{\partial c_{i,t-}} V(y_t, c_{i,t-}) = -w_i w_{F,i} \beta_{i,t} u'(c_{t-}) \tag{106}$$

when the planner decreases agent i's F-consumption. Thus, there exists an inaction interval $[\underline{c}_{i,t}y_t, \bar{c}_{i,t}y_t]$ such that the agent's consumption is not adjusted if her consumption at $t-$ is inside the interval and adjusted to the nearest boundary if it is outside the interval.

The economy can be decentralized to obtain a competitive equilibrium in a standard way (Duffie and Huang (1985), Wang (1996)).

The SDF is determined by the marginal utility of A-consumption and takes the following form:

$$\mathcal{H}_t = e^{-\delta t}(y_t - c_{F,t})^{-\gamma} = e^{-\delta t}\mathcal{A}_t^{-\gamma} y_t^{-\gamma}, \tag{107}$$

where $c_{F,t}$ is the aggregate F-consumption, i.e., $c_{F,t} = \sum_{i=1}^m c_{i,t}$ and $\mathcal{A}_t$ is the aggregate A-share.

As the number m of averse agents increases, the effect of each individual averse agent becomes smaller and becomes negligible in the limit if the weight of each agent shrinks to zero. If an agent's weight remains positive, then the agent's consumption adjustment can have a significant effect on the SDF even in the limit. This can represent a real-world situation in which a large number of agents, whose aggregate consumption comprises a non-trivial fraction of output, simultaneously adjusts consumption.

We will now consider the dynamics of aggregate F-consumption $c_{F,t}$ in the extended model. Suppose that $c_{F,t}$ is a semimartingale. It has a unique Doob-Meyer decomposition:

$$c_{F,t} = c_{F,0} + A_t + M_t, \tag{108}$$

where A_t is of bounded variation and M_t is a local martingale. Suppose that M_t is has a quadratic variation, which is absolutely continuous with respect to the Lebesgue measure. Then, by the martingale representation theorem (Theorem 4.2, Karatzas and Shreve (1998)), we know that for some $\sigma_{M,t}$

$$M_t = \int_0^t \sigma_{M,t} dB_t,$$

and for some $\mu_{A,t}$ and K_t

$$A_t = \int_0^t \mu_{A,t} dt + K_t,$$

where K_t is a singular process, i.e., has derivative equal to 0 a.e. with respect to the Lebesgue measure. Let $\mu_{F,t} = \frac{\mu_{A,t}}{c_{F,t}}$, $\sigma_{F,t} = \frac{\sigma_{M,t}}{c_{F,t}}$, and $\Delta c_{F,t} = \frac{dK_t}{c_{F,t}}$, we arrive at (31).

I Proof of Propositions 4.1 and 5.1

We will prove Proposition 5.1. Proposition 4.1 follows from Proposition 5.1.

Since $\mathcal{H}_t = e^{-\delta t}u'(c_{A,t}) = e^{-\delta t}(y_t - c_{F,t})^{-\gamma}$, the generalized Itô's lemma implies that

$$\begin{aligned} d\mathcal{H}_t =& -\delta\mathcal{H}_t dt - \gamma e^{-\delta t}(y_t - c_{F,t})^{-\gamma-1}dy_t + \gamma e^{-\delta t}(y_t - c_{F,t})^{-\gamma-1}dc_{F,t} + \frac{\gamma(\gamma+1)}{2}e^{-\delta t}(y_t - c_{F,t})^{-\gamma-2}(dy_t)^2 \\ &+ \frac{\gamma(\gamma+1)}{2}e^{-\delta t}(y_t - c_{F,t})^{-\gamma-2}(dc_{F,t})^2 - \gamma(\gamma+1)(y_t - c_{F,t})^{-\gamma-2}(dy_t)(dc_{F,t}) + \gamma\mathcal{H}_t\frac{\Delta c_{F,t}}{y_t - c_{F,t}}. \end{aligned} \tag{109}$$

It follows that

$$\frac{d\mathcal{H}_t}{\mathcal{H}_t} = -\left(\delta + \frac{\gamma\mu}{\mathcal{A}_t} - \frac{\gamma\mu_{F,t}(1-\mathcal{A}_t)}{\mathcal{A}_t} - \frac{\gamma(\gamma+1)}{2}\frac{(\sigma - \sigma_{F,t}(1-\mathcal{A}_t))^2}{\mathcal{A}_t^2}\right)dt + \gamma\mathcal{H}_t\frac{\Delta c_{F,t}}{y_t - c_{F,t}} - \gamma\left(\frac{\sigma}{\mathcal{A}_t} - \frac{\sigma_{F,t}(1-\mathcal{A}_t)}{\mathcal{A}_t}\right)dB_t$$

Therefore, matching the terms, the market price of risk θ_t is the coefficient of $\mathcal{H}_t dB_t$ and the instantaneous bond return is given by $r_t dt + \Delta G_t$ as in Proposition 5.1.

J Lucas Economy

We first consider the Lucas economy in which there is only adjustable consumption in the heterogeneous consumption interpretation of the model. It is a representative agent economy studied by Lucas (1978). In this economy the SDF $\mathcal{H}_t$ is given by

$$\mathcal{H}_t = e^{-\delta t} y_t^{-\gamma}. \tag{110}$$

The risk-free rate r_t is constant and given by

$$r_t = \rho \equiv \delta + \gamma\mu - \frac{1}{2}\gamma(1+\gamma)\sigma^2.$$

We will assume $\rho > 0$.

The market price of risk is constant and given by

$$\theta = \gamma\sigma.$$

The stock price can be calculated as

$$S_t = \frac{1}{\rho_s} y_t,$$

where $\rho_s = \delta - (1-\gamma)\mu + \frac{1}{2}\gamma(1-\gamma)\sigma^2$. The risk premium ρ_E of the holding period return on the stock is given by

$$\mathbb{E}_t\left[\frac{dS_t + y_t dt}{S_t}/dt\right] - r_t = \mu + \rho_s - \rho = \gamma\sigma^2.$$

The prices $P(t,T), S(t,T)$ of the zero coupon bond and the equity strip with maturity T are as follows:

$$P(t,T) = e^{-\rho(T-t)}, \ S(t,T) = e^{-\rho_s(T-t)} y_t. \tag{111}$$

Consequently, the zero coupon bond is risk-free and the equity strip has the risk premium

$$\mathbb{E}_t\left[\frac{dS(t,T)}{S(t,T)}/dt\right] - r_t = \mu + \rho_s - \rho = \gamma\sigma^2. \tag{112}$$

That is, the stock and the equity strip have the same risk exposure and the same constant risk premium equal to $\gamma\sigma^2$ for every maturity $T > t$. The term structure of risk and return trade-off is flat in the Lucas economy.