The Sources of Risk in Credit Portfolio

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30th April 2021

Abstract

This paper proposes a new method that identifies risk sources of credit portfolio's default clustering as macroeconomic factors, default contagion (asset correlation), and frailty effect under the Basel regulatory framework. Our model estimates the three time-varying risk sources and their contributions using Hoeffding decomposition.

Our empirical results for the U.S. aggregated loan sectors find that the default clustering in each loan portfolio strengthens during economic downturns. The risk contributions to default clustering are large in order of macroeconomic source, asset correlation, and frailty effects. The dynamics of risk sources show different in affecting sectors and affected sectors for each crisis. Furthermore, using risk sources contribution analysis by cross-sector, we find that the subprime mortgage crisis is a systemic event that affects the entire banking system through the macroeconomic source, while the dotcom bubble crisis is a locally sectoral systemic event. From these results, we have checked the possibilities that the excess default clustering can be explained using the time-varying risk sources.

Keywords: Default clustering, Frailty effect, Asset-correlation, Credit portfolio risk, Time-varying risk parameters, Systamatic risk factor, ASRF model, LHP assumption.

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1 Introduction

The default rate depends on the economic conditions as widely acknowledged in the field of credit risk (see Pesaran et al. (2006), Koopman et al. (2012), and Bonfim (2009)). The default clustering during the economic crisis, as depicted in Figure 1, is considered an origin of a fat tail distribution of unconditional credit portfolio loss and causes more difficulty in credit risk modeling. Therefore, measuring the default clustering and identifying their sources are crucial for credit portfolio management at financial institutions, as well as the systemic financial risk management at regulators. This paper suggests a new method that identifies time-varying risk sources of credit portfolio's default clustering as macroeconomic factors, default contagion (asset correlation), and frailty effect based on the regulatory framework.

[Figure 1 is here.]

The regulatory framework has been used the Asymptotic Single Risk Factor (ASRF) model based on Merton (1974) where an obligor's default event occurs when its asset value below down its threshold at debt maturity. The asset value is assumed to be determined by two components: one is the systematic factor and the other is obligor-specific factor. The sensitivity to the systematic factor (asset correlation) is assumed to be constant as a risk parameter.

The single factor assumption on the ASRF model is mitigated by including various observable covariates (Rösch (2003), Crook and Bellotti (2010), and Hamerle et al. (2003) for macro-economic factors and historical default rates) and frailties (Duffie et al. (2009) and Koopman et al. (2011)[1] for common frailty, Kwon and Lee (2018), Jiménez and Mencia (2009) and Lee and Poon (2014) for industry-specific frailty) to explain and predict the real default rates of corporates and credit portfolios point-in-time.

The previous models have been mainly focused on decomposing the common systematic factor into the multiple factors to capture the default clustering, not considering the variation of sensitivity to the systematic risk factor. In the credit portfolio model based on the Large Homogeneous Portfolio (LHP) assumption (see Vasicek (1991), Vasicek (2002), Gordy (2003), and Committee et al. (2005)), the sensitivity to the systematic factor also implies a measure of the co-movement among different obligor's asset values and could be entitled the measure of the contagion among the obligors within a portfolio. Azizpour et al. (2018) shows that the contribution of firm-by-firm contagion more increases during default clustering episode periods in economic crisis. In addition to this, there are many empirical studies for procyclical evidence for asset correlation with economic conditions. (see Lee et al. (2011), Botha and van Vuuren (2010), Siarka (2014), and Stoffberg and van Vuuren (2016)). However, the time-varying asset correlation has not been applied to credit portfolio model for regulatory purposes[2].

[1]Koopman et al. (2011) attempts to explain the default clustering using more than 100 macro variables and firm-specific variables by 112 groups (industry, age of firm, and credit rating) for testing omitted variable problem.

[2]Regulatory authorities are regulating individual financial institutions through the consevative guidelines

In this paper, through the time-varying modelling for the two risk parameters, we suggest a method that could identify risk sources at each point-in-time for the dynamics of the portfolio loss depending on economic conditions. Firstly, the dynamic default threshold consisting of only lagged macroeconomic covariates is used to model the expected value of the portfolio default rate. From this model, we can estimate the expectation of portfolio loss given macroeconomic information. It is a conditional (only macro information) mean of portfolio loss at time t, simultaneously it is a value that stands for the source of default clustering from macroeconomic conditions. This approach is similar to the dynamic default threshold model by Rösch (2003), Crook and Bellotti (2010), and Hamerle et al. (2003), but differs in use of only lagged macro covariates based on LHP assumption. Secondly, the bivariate time-varying copula model by Patton (2006) is modified to estimate the univariate time-varying asset correlation for the comovement among the obligors' asset within the portfolio. This risk source could express the excess default clustering after controlling the expected portfolio default level from the macroeconomic source. Thirdly, the frailty effect as the other risk source is calculated using the estimated two risk sources and the realized portfolio loss. In our time-varying risk sources model, we examine the possibilities that the excess default clustering could be expressed using the source from asset correlation in the economic crisis. Lastly, our paper suggests a simulation method to calculate contributions of each estimated risk source for point-in-time portfolio losses employing the Hoeffiding decomposition using gaussian copula correlation between them. [3]

As an empirical study, we estimate the time-varying risk sources for aggregate loan sector level portfolio loss of the U.S. commercial banking system using the quarterly charge-off rates. We find that losses of each sector portfolio could be predicted by different macro covariate. For example, the lagged macro covariates used to explain the mortgage sector expected loss are clearly distinguished compare to other sectors' them. The time-varying asset correlations show a lower value than static assumption asset correlation during normal economic conditions in all sectors. However, it shows increasing rapidly in the crisis periods, then consisting a high level for a while even though macroeconomic condition recovered. Each crisis period has different the most volatile sectors for asset correlation by the risk originated sectors. Furthermore, we have observed that the channels as risk sources may reveal differently for each crisis by the loss contribution analysis results aspect of cross-sectoral risk contagion. Lastly, we have checked the utilization for time-varying risk sources through a comparison of the frailty effects which are estimated based on various assumptions for risk parameters.

The contributions of this paper are fourfolds. Firstly, our methods propose to identify

in Internal rating-based credit capital by use to long-term stressed default threshold.

[3]Although our model does not set distinguished models for the contagion and the frailty exactly as in Azizpour et al. (2018) which used the past default information using default timing data for corporate obligors, it can reflect the excess default clustering in observation time t as well as past default level information in within the portfolio.

risk sources of the default clustering at each point-in-time. Through the time-varying risk parameters setting, we could decompose the portfolio loss into the dynamics effect from macroeconomic condition, contagion among obligors within portfolio, and frailty. By separating the mixed effect of the macroeconomic covariates' dynamics and the asset correlation effects in previous multi-factor approaches, each risk source of the portfolio loss could be more accurately and independently identified. Secondly, in our model, most of the frailty effect known as a latent factor could be expressed intuitively by the dynamics of asset correlation source. Our asset correlation model has similar characteristics of the frailty factor as time-varying and mean-reverting because it expresses uncertainty that is unexplained excess part of loss above the expectation from given information. Thirdly, our methods could be useful ways to capture portfolio risk sources and their contributions for practitioners as not only portfolio managers but also regulators. They might build strategies for managing credit risks on a source-by-source basis to ensure the stability of each portfolio's as well as entire financial systems. It can also minimize the model risk from not only static risk parameter assumption but also the lack of data in the rolling window method[4]. Finally, our proposed methods are simple and easy tools without additional data and assumptions under the current standard credit risk management process based on the various constraints of the regulatory frame nevertheless adopt the realistic assumption for default threshold and asset correlation. To the best of our knowledge, our methods are the first approach to employ all time-varying parameters for the portfolio credit risk model.

The remainder of this paper is organized as follows. Section 2 develops the time-varying loss distribution using time-varying sources depending on economic conditions to capture the default clustering. In the empirical analysis of Section 3 using the loan sector level aggregate charge-off rates of the U.S. banking system, each portfolio risk source is decomposed and its contribution to the portfolio loss is measured. Section 4 conclues with comments.

[4]Many studies on the dynamics of asset correlation have been conducted to overcome the limitation as a static parameter. All most of studies for asset correlation are estimated by dividing the entire data into parts, such as the recession dummy variables or the rolling window method. (See Lee et al. (2011); Botha and van Vuuren (2010); Siarka (2014); Stoffberg and van Vuuren (2016)) However, these approaches also have problems with the following two aspects. Firstly, These are able to underestimate the asset correlation that is realized in the crisis window since the effect of reducing volatility in the estimation using the average value. Secondly, rolling-window estimates requires sufficient data to be obtained within each estimation period for reducing model risk, but the default data is not enough since its observation frequency is quarterly or be more.

2 Methodology

The famous models for measuring obligor level default risk are the structural model based on the option pricing formula and the intensity model based on the survival analysis. In this section, we briefly review the individual obligor level default and drive the portfolio level loss model under Large Homogeneous Portfolio (LHP) assumptions using the structural model based on the Basel regulation. And, we identify the risk sources for the portfolio loss distribution and then add the time-varying modeling for them to drive the time-varying portfolio loss function.

2.1 Basel's portfolio credit risk model

As a well-known structural model by Merton (1974), individual obligor's default event occurs when the asset return becomes less than the value of unexpired liabilities during a certain period of time. That means the net asset value is negative. For obligor level default probability model, let V_i and h_i be asset return and default threshold for borrower i, respectively. Assume that the distribution of obligor's asset return is standard normal distribution, the unconditional default probability of obligor i is given by

$$p(y_i = 1) = p(V_i < h_i) = \Phi(h_i) \tag{1}$$

where y_i denotes the default indicator for obligor i, taking either the value of one for default or the value of zero for non-default. And Φ is a cumulative standard normal distribution applied for all time periods. As shown Eq. (1), the unconditional default probability of an individual obligor can be expressed as a function of the default threshold of the corresponding borrower. Let obligor i's standard asset return (or creditworthiness) V_i be a function of the single systematic risk factor F and the idiosyncratic risk ε_i.

$$V_i = \sqrt{\rho_i}F + \sqrt{1-\rho_i}\varepsilon_i, \qquad for \qquad i = 1, \cdots, N \tag{2}$$

where $F \sim N(0,1)$ and $\varepsilon_i \sim N(0,1)$, then $V_i \sim N(0,1)$. F and ε_i are assumed to be independent for all i and $Cov(\varepsilon_i, \varepsilon_j) = 0$ when $i \neq j$. Then $\sqrt{\rho_i}$ means the obligor's sensitivity to the common systematic factor as a linear correlation between V_i and F. The bigger the asset correlation $\sqrt{\rho_i}$, the asset returns are more affected by the common systematic factor F and less the obligor specific idiosyncratic risk ε_i. The obligor i's conditional credit risk depending on economic conditions are dominated by the only common systematic factor F that assumed standard normal distribution because asset correlation ρ_i and default threshold h_i are assumed static values all time-periods under the one-factor model framework (see Hamerle et al. (2003)). F is a factor that affects all borrowers $i = 1, \cdots, N$ commonly and is assumed can be diversified away. We can write that the conditional default probability of obligor i is given by

$$p(y_i = 1|f) = \mathbb{P}(V_i < h_i|F = f) = \Phi\left(\frac{h_i - \sqrt{\rho_i}f}{\sqrt{1-\rho_i}}\right) \tag{3}$$

where f is realized systematic risk factor. Both the structural model as well as the default intensity model on the obligor level attempt to break down systematic factor F into observable macroeconomic or individual-specific covariates, which are called the multi-factor approach models. And the models for exceeding default clustering use the frailty effect to explain the excess default concentration phenomenon even if control these multi-factors.(expression by koopman 2011 : even after controlling

For portfolio level credit risk model, Vasicek (1991) and Gordy (2003) proposed the Asymptotic Single Risk Factor (ASRF) model that approximates credit portfolio loss distribution under the following assumptions known as large homogeneous portfolio (LHP). Assumption 1. Homogeneity: Individual assets within the portfolio share the same value of risk parameters such as asset correlation ρ and default threshold h. Assumption 2. Fine grained and Large portfolio: Each loan size is so evenly distributed that individual loans do not dominate the portfolio. And, there are countless (infinite) borrowers in the portfolio, so there is no contribution of individual loans on the entire portfolio. This asymptotic model is based on large number theory. This approach assumes that all obligors within each portfolio are affected solely by a single common risk factor F and their idiosyncratic risks are diversified away.

We can omit subscript i in Eq. (3) on LHP assumption for simplicity and write the each obligors' conditional default probability within portfolio g as

$$p(y|f) = \Phi\left(\frac{h_g - \sqrt{\rho_g}f}{\sqrt{1-\rho_g}}\right) \tag{4}$$

where the default threshold h_g and the sensitivity $\sqrt{\rho_g}$ are identical risk components in each portfolio. This model assumes that all obligors within portfolio g are adopted the same default probability under LHP assumptions. Thus, the asset correlation between i and j is $Corr(V_i, V_j) = \rho$ when $i, j \in g$ and $i \neq j$, because $\rho_i = \rho_j = \rho$. We denote for the portfolio of size n, it's default rate L_n as

$$L_n = \frac{1}{n}\sum_{i=1}^{n} I_{(V_i<h_g)} \tag{5}$$

where $I_{(V_i<h_g)}$ is the default indicator under the value 1 if $V_i < h_g$, and 0 otherwise. Thus default rate L_n on portfolio g converses to obligors' conditional default probability $p(y|f)$ under LHP assumptions by law of large numbers as,

$$L_g = \lim_{n\to\infty} L_n = p(y|f) \tag{6}$$

.

Since the common systematic factor F is random variable on $F \sim N(0,1)$ assumption, the cumulative distribution function of unconditional portfolio loss based on LHP assumption can drive as

$$\mathbb{F}(\ell_g; h_g, \rho_g) = \mathbb{P}[L_g < \ell_g; h_g, \rho_g] = 1 - \Phi\left(\frac{h_g - \sqrt{1-\rho_g}\Phi^{-1}(\ell_g)}{\sqrt{\rho_g}}\right) \tag{7}$$

where ℓ_g is a realized default rate on portfolio g. And $\mathbb{F}(\ell_g; h_g, \rho_g)$ is a function with the identical parameters of ρ_g , h_g and realized value ℓ_g. Not only the Basel framework but also all most credit risk models apply the static assumption for shared identical parameters h_g and ρ_g within portfolio g regardless of economic condition and time. This portfolio credit risk model has been adopted in the Internal rating-based(IRB) approach for Basel II regulatory capital calculation and standard credit risk managing process for financial institutions. This paper also calls Eq. (4) and Eq. (7) as "Static model" and compares with the "Time-varying model" that we will suggest in the next section.

2.2 Time-varying risk sources

Among the components of Eq. (7), asset correlation ρ_g and default threshold h_g are known as important parameters in determining the shape of portfolio unconditional loss distribution (see Gordy (2003) and Lee et al. (2020)). Especially, asset correlation ρ_g, which is a sensitivity to systematic risk F is a crucial parameter in determining the tail shape when charging the regulatory capital against financial institutions for securing financial system stability. Nevertheless, due to static assumptions on the credit risk model parameters, these models can not reflect a comovement of assets according to economic conditions.

Our paper defines the two time-varying risk sources as asset correlation $\rho_{g,t}$ and default threshold $h_{g,t}$ to overcome static parameter assumptions using the average or conservative value. For these purposes, we define the time-varying default threshold model that expected default rate level implied in macroeconomic information. And, we capture the excess default clustering above the expected default rate level using the time-varying asset correlation $\rho_{g,t}$ within the credit portfolio. Moreover, the frailty effect $f_{g,t}$ as the latent variable estimates the residual default clustering even if remains controlled by two type time-varying risk sources. Our approach differs from the prior models in the following terms. Firstly, all of the static parameters for portfolio credit risk are assumed by our time-varying risk sources. All most dynamic credit models[5] assume the time-varying variable for the subset of parameters. But

[5]Hamerle et al. (2003) investigate the absolute value of the static asset correlation is sharply reduced in the model that assumes the dynamic default threshold compared to the model that assumes static it. And, dynamic asset correlation models(Lee et al. (2011); Botha and van Vuuren (2010); Siarka (2014); Stoffberg and van Vuuren (2016)) are used to estimate the unconditional long-term default threshold.

our model could identify all risk sources consisting of portfolio loss distribution Eq. (8) at point-in-time. Secondly, by modeling for time-varying asset correlation within the portfolio, most of the explanatory effect for default clustering as the frailty effect can be decomposed to asset correlation dynamics, and the remaining default clustering could be expressed to frailty effect.

As a result of time-varying risk sources assumption, we can modify Eq. (7) to time-varying cumulative distribution function of portfolio loss as,

$$\mathbb{F}(\ell_g; h_g, \rho_g) = 1 - \Phi\left(\frac{h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})}{\sqrt{\rho_{g,t}}}\right) \tag{8}$$

. Because this function consists of all time-varying parameters (risk sources), if we could know their estimates, then we can estimate dynamics for unconditional loss distribution of credit portfolio point-in-time dependimg on macroeconomic conditions.

2.2.1 Time-varying macro source

The Basel committee regulates financial institutions' risk management system under internal rating based (IRB) rules for counterparty's default rate. When estimating unconditional default probability (UPD) and calibrating credit rating on these rules, financial institutions should have to use sufficiently long-term stressed default rate to cover the downturn economic cycle under the Through The Cycle (TTC) method. This approach is only a conservative criterion for credit rating stability and regulatory compliance, but there is a limit to actually reflecting the risk dynamics of the portfolio by economic conditions. For example, although the credit rating grade covers downturn periods by this approach, but it does not reflect the volatility according to the economic cycles within grades (see Catarineu-Rabell et al. (2005) and Kashyap et al. (2004)).

Hamerle et al. (2003) suggests the dynamic default threshold model to overcome the shortcomming of static default threshold and systematic factor F on independent and identically standard normal distributed assumption. They point out that systematic factor F is not an iid random variable, but an affected value for time $t-1$ macroeconomic status. Thus, Rösch (2003) as well as Crook and Bellotti (2010) suggest the setting of the default threshold as a model affected by observable systematic factor proxies at time $t-1$ (e.g. lagged macroeconomic and obligor specific covariates) unlike conventional static models. These are called point-in-time (PIT) method models that express the dynamics and forecast the next period for default rate.

Most PIT models focus on modeling the default rate of individual borrowers level as corporate debt. But, our model targets an aggregate portfolio level loss that can be used for the bank's portfolio managers and regulators. In this paper, we do not consider individual obligors specific covariates (idiosyncratic factor) within portfolio because of the portfolio loss distribution based on LHP assumptions in Section 2.1. And, the characteristics of the distribution on credit portfolio losses may differ depending on the sector of the loan classified by the financial institutions. So, our time-varying default threshold model defines by the type of portfolio, respectively and each model selects macroeconomic covariates and their lagging time τ differently.

To model the expected default rate according to macroeconomic conditions, we assume that default rate of the portfolio g are independent given only the observable time-lagged macro covariates $z_{k,t-\tau}$ at time t. Under this assumption, we can denote the time-varying default threshold model $h_{g,t}$ as

$$h_{g,t} = \beta_0 + \sum_{k=1}^{q_z} z_{k,t-\tau}\beta_k^z \tag{9}$$

where $z_{k,t-\tau}$ is the $k-th$ observable macroeconomic variable at time $t-\tau$ for $k = 1, 2, \cdots, q_z$, and intercept β_0, and parameters β_k^z are the sensitivities of portfolio g to common macroeconomic covariates. We denote a vector of observable common macro covariates as $\boldsymbol{z}_{t-\tau} = (z_{1,t-\tau}, z_{2,t-\tau}, \cdots, z_{q_z,t-\tau})'$. Based on this modeling property, we denote default threshold for portfolio g using Eq. (1) as

$$\begin{aligned} \mathbb{P}(V_{g,t} < h_{g,t}|\boldsymbol{z}_{t-\tau}) &= \Phi\left(\beta_0 + \textstyle\sum_{k=1}^{q_z} z_{k,t-\tau}\beta_k^z\right) \\ &= \Phi\left(h_{g,t}\right). \end{aligned} \tag{10}$$

The macro economic covariates are time-lagged in dynamic default threshold model Eq. (9) as each of the portfolios are generally foreclosed[6] after a certain period of borrower distress. Hence, macroeconomic covariates are expected to lead portfolio defaults by time τ. This property is the great practical importance, as it implies that portfolio's default rate can be forecasted using well-given macroeconomic information that is available at the time $t-\tau$. Forecasting the credit risk of the individual obligors may be an important process for the monitoring of loans or the early warning of borrowers' default event. Furthermore, earlier perceiving the dynamics of the portfolio's loss is a more critical issue for the portfolio managers of financial institutions and supervisors to accomplish financial system stability. This paper is interested in four quarters relation of default threshold and time lagged macro covariates as $\tau = 1, 2, 3, 4$. This model examines the dynamics of expected default rate level $\Phi(h_{g,t})$ for the portfolio from the observable macroeconomic information.

Each portfolio time-varying default threshold models estimate by following the three-step tests. Step 1 : After selecting the first candidate covariates through the previous research[7], in order to capture the time lagged relationship between each economic covariate $z_{k,t-\tau}$ and

[6] Foreclosure is the default criterion in the empirical analysis.

[7] More detail studies for macroeconomic covariates to relate credit risk by loan type are in section 3.1.

the default threshold $h_{g,t} = \Phi^{-1}(\ell_{g,t})$ on portfolio g. We examine the second candidate covariates are selected by considering the significance of cross-correlation statistics and an explanation[8] of the economic model simultaneously. Step 2 : The multicollinearity and model explanatory power check for temporal multivariate regression analysis using macroeconomic covariates selected Step 1. In this step, we choose the third candidate for the final model. Step 3 : The time-varying default threshold model and the time-varying asset correlation model in the section 2.2.2 are optimized simultaneously when estimating the final time-varying loss distribution Eq. (8).

2.2.2 Time-varying asset correlation

We define the $\rho_{g,t}$ is the time-varying asset correlation of portfolio g at time t for $t = 1, 2, \cdots, T$ as,

$$\rho_{g,t} = \tilde{\Lambda}\left(\alpha_{g,0} + \alpha_{g,1}\rho_{g,t-1} + \alpha_{g,2}\frac{1}{S}\sum_{s=1}^{S}\left(\Phi^{-1}\left(u_{g,t-s}\right)\right)^2\right) \tag{11}$$

where $\tilde{\Lambda}(x) = (1 + e^{-kx})^{-1}$, $k > 0$, $-\infty < \alpha_0 < \infty$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$. The logistic transformation is intended to limit 0 to 1 the bounds of estimates in the time-varying asset correlation model. The univariate variance term $u_{g,t-s}$ is the estimated portfolio loss $\mathbb{F}(\ell_{g,t-s}, h_{g,t-s}, \rho_{g,t-s})$ on sector g at time $t - s$ given $h_{g,t-s|t-s-\tau}$ and $\rho_{g,t-s}$. We modify the original specification of Patton (2006) that used the covariance terms between high-frequency exchange rates for time-varying conditional dependence, but our model is different in using the univariate variance term instead of them. Our time-varying asset correlation model Eq. (11) can modify the multi-variate covariance term to the univariate variance under the homogeneity assumption of assets within the portfolio under the LHP assumption in section 2.1. This setup is similar to the GARCH(S,1) type process for the volatility model. Our model assumes that asset correlation at time t consists of two terms. First, through the auto-correlation term of asset correlation just before (time $t - 1$), it intends to reflect the long-term memory of the average relation among assets in the loan portfolio. This approach is similar to Duffie et al. (2009) in terms of the AR(1) process assuming for the frailty to capture the default clustering. The second term, the moving average of the univariate volatility from the estimated endogenous loss distribution at time $t - s$ is used to model the persistence of the impact of portfolio loss due to the economic fluctuations. This is the setting of a model to reflect the impact of short-term uncertainty in the loss distribution inherent in the asset correlation and macroeconomic covariates during the $t - s$ period using the GARCH type model. The impact persistence of these shocks also varies depending on the loan type by portfolio, so it is set up

[8] It means that our selective rule is based on the sign of the correlation between credit risk and macroeconomic covariate in previous literature.

respectively.

We can represent cumulative distribution function for time-varying portfolio loss rate in Eq. (8) with time-varying default threshold in Eq. (9) and time-varying asset correlation in Eq. (11). And then, using the realized portfolio g's default rate $\ell_{g,t}$ at time t, we can estimate parameters in the time-varying models from the probability density function (PDF) of Eq. (8) by Maximum Likelihood Estimation (MLE) methods.

More detail for the PDF and MLE methods for time-varying or static portfolio loss are in Appendix 2. We can write the product of likelihood at each time t as,

$$\max_{\boldsymbol{\theta}} \prod_{t=1}^{N} \sqrt{\frac{1-\rho_{g,t}}{\rho_{g,t}}} \cdot exp\left[\frac{1}{2\rho_{g,t}}\left(h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \tag{12}$$

where $\boldsymbol{\theta} = (\boldsymbol{\alpha_g}, \boldsymbol{\beta_g}^z)'$ are the time-varying asset correlation model parameters $\boldsymbol{\alpha_g} = (\alpha_{g,0}, \alpha_{g,1}, \alpha_{g,2})'$ in Eq. (11) and the time-varying default threshold parameters $\boldsymbol{\beta_g}^z = (\beta_0, \beta_1^z, \beta_2^z, \cdots, \beta_k^z)'$ in Eq. (9) from the lagged economic coavriates. If we know the time-varying models' parameters $\boldsymbol{\theta}$, then we can get the estimates of time-varying default threshold $h_{g,t}$ and asset correlation $\rho_{g,t}$ point-in-time. This means we are able to examine the dynamics of the unconditional loss distribution $\mathbb{F}(\ell_{g,t}, h_{g,t}, \rho_{g,t})$ in Eq. (8) at each observation time t.

2.3 Risk sources decompositon and contribution

2.3.1 Risk sources decompositon

The fluctuation of portfolio loss under the static parameters assumption model Eq. (4) is dominated by systematic common one factor. But our time-varying models in previous section 2.2, that decompose various risk sources dynamics as the expected default rate level $\Phi^{-1}(h_{g,t})$ from macroeconomic information and the asset correlation $\rho_{g,t}$ from contagion effect. And, we define the residual effect of systematic common factor as "frailty effect" $f_{g,t}$ that even controlling the two estimated time-varying risk sources.

From conditional portfolio default rate in Eq. (4) given time-varying risk sources and realized default rate at time t, we can summarize for pure frailty effect $f_{g,t}^{TV}$ on portfolio g at time t as

$$f_{g,t}^{TV} = \frac{h_{g,t} - \sqrt{1-\rho_{g,t}} \cdot \Phi^{-1}(\ell_{g,t})}{\sqrt{\rho_{g,t}}} \tag{13}$$

Apply the above approach Eq.(13) to static model Eq. (4), we can estimate the frailty effect f_g^{St} under static risk sources assumption similar to frailty effect in previous literature (Lee and Poon (2014),Duffie et al. (2009), and Koopman et al. (2011). etc.). In the empirical study in Section 3.4, we compare the contributions of f_g^{St}, $f_{g,t}^{TV}$, and another frailty effect under partly static parameter assumption to check the adequacy of time-varying risk sources model for default clustering.

Despite the effort attempt to describe the default clustering by many macroeconomic or obligor specific covariates in the previous static parameter assumption models, but remain the significant frailty effect for default clustering as an unexplainable latent factor. Our estimated pure frailty $f_{g,t}^{TV}$ remains latent factor characteristic yet, but a significant part of the frailty effect f_{g}^{St} on static assumption has been removed by the asset correlation dynamics.

2.3.2 Risk sources contribution

If the portfolio managers could identify actual risk sources and measure their contributions exactly, they could control the credit risk as hedging or other portfolio rebalancing strategies[9]. In this section, we propose the Hoeffding decomposition method using the conditional copula simulation to estimate the contribution for portfolio losses from estimated risk sources in our time-varying model. The Copula function[10]widely is used in financial applications[11] for decoupling a multi-variate joint distribution to marginal distributions and their dependence structure. And the Hoeffding decomposition[12] is a familiar method decomposing for factor contributions in risk management fields. In particular, Rosen and Saunders (2010) as well as Lee and Poon (2014) show that the portfolio loss as a random variable can be decomposed as the sum of expected loss given all subsets of systematic risk factors (e.g. macroeconomic or frailty). They measure the contributions of the loss distribution for each factor using a linear additive multi-factor model as extended for systematic factors. However, Tasche (2008); Cherny et al. (2010) show that the Hoeffding decomposition can be used to decompose the effects of nonlinear factors.

We present the simple example case for decomposing two factors F_1 and F_2 to understand in the Hoeffding decomposition method. We can write the portfolio loss $L = H(F_1, F_2)$ as

$$\begin{aligned} L &= E[L|\cdot] \\ &+(E[L|F_1] - E[L|\cdot]) + (E[L|F_2] - E[L|\cdot]) \\ &+(E[L|F_1, F_2] - (E[L|F_1] - E[L|\cdot]) - (E[L|F_2] - E[L|\cdot]) - E[L|\cdot]) \end{aligned} \tag{14}$$

where the $PD_{Base} = E[L|\cdot]$ term in first row Eq. (14) is the base level of default rate (unconditional expected loss level as constant) without any risk factors. The $E[L|F_1]$ and $E[L|F_2]$ terms in second row denote the expected portfolio loss $E[L]$ given factor F_1 and F_2, respectively. And the $RC^{F_k} = (E[L|F_k] - E[L|\cdot])$ operations owing to estimate pure

[9] Asset allocation, Risk budgeting, etc.

[10] The full description and mathematical backgrounds are in Nelsen (2007) and Catarineu-Rabell et al. (2005).

[11] The various applications of copula function are described in McNeil et al. (2015),Patton (2006), and Lee and Yang (2019).

[12] The original development methodology is described in Van der Vaart (2000).

risk contribution from factor F_k controlled the base default rate level PD_{Base}. The term of last row represents the residual risk contribution RC^{F_1,F_2} controlled individual risk factor contribution RC^{F_k} and base default rate level PD_{Base}. This means the pure expected loss given comovements in the factors F_1 and F_2.

Our decomposing method extends Eq. (14) to three risk sources for within portfolio risk contribution. Among them, we are interested in the following four terms in order to capture the pure contribution of each source at time t and the base expected loss level on portfolio g. The conditional expected losses given the risk sources within portfolio denote the risk contributions as

$$\begin{aligned} PD_{g,Base}: &\quad E[L_{g,t}|\cdot] \\ RC_{g,t}^{\Phi(h_{g,t})}: &\quad (E[L_{g,t}|\Phi(h_{g,t})] - E[L_{g,t}|\cdot]) \\ RC_{g,t}^{\rho_{g,t}}: &\quad (E[L_{g,t}|\rho_{g,t}] - E[L_{g,t}|\cdot]) \\ RC_{g,t}^{f_{g,t}}: &\quad (E[L_{g,t}|f_{g,t}] - E[L_{g,t}|\cdot]) \end{aligned} \tag{15}$$

The $PD_{g,Base} = E[L_{g,t}|\cdot]$ term represents the base expected loss level in portfolio g. This term means the level of unconditional expected loss without any risk sources as constant value. And, the other terms $RC_{g,t}^{\Phi(h_{g,t})}$, $RC_{g,t}^{\rho_{g,t}}$, and $RC_{g,t}^{f_{g,t}}$ in Eq. (15) are denoted the pure risk contributions even controlling $PD_{g,Base}$ on portfolio g at time t from each risk source $\Phi(h_{g,t})$, $\rho_{g,t}$, and $f_{g,t}$, respectively.

However, in estimating process for the risk contributions, we can not know the joint density of the portfolio loss given estimated risk sources at time t since the only one observation point of them at each time. Thus, we suggest the conditional copula simulation method that estimates the joint distribution by the kernel density based on the empirical dependence structure between portfolio loss and estimated risk sources. For this purpose, we apply the Novosyolov (2017) method for conditional distribution for portfolio loss given risk sources at time t. The expected values are calculated for the conditional loss distribution using 1 million Monte Carlo simulation reflecting the Gaussian copula dependence structure from the empirical kernel joint density. Through this process, we estimate the risk contributions of the portfolio loss from each risk source as presented in the empirical study of Section 3.3.1.

Moreover, we extend the within portfolio method Eq. (15) to the cross-sectional dimension to estimate the risk contagion effect by simulating the expected loss on portfolio g given the causative sector's risk sources. For this purpose, we define the pure risk contributions as $RC_{g,t}^{\Phi(h_{c,t})}$, $RC_{g,t}^{\rho_{c,t}}$, and $RC_{g,t}^{f_{c,t}}$ that are expected loss $E[L_{g,t}]$ on portfolio g controlled $PD_{g,Base}$ given the risk sources of causative sector c as $\Phi(h_{cause,t})$, $\rho_{cause,t}$, and $f_{cause,t}$, respectively,

$$\begin{aligned} RC_{g,t}^{\Phi(h_{c,t})}: &\quad (E[L_{g,t}|\Phi(h_{c,t})] - E[L_{g,t}|\cdot]) \\ RC_{g,t}^{\rho_{c,t}}: &\quad (E[L_{g,t}|\rho_{c,t}] - E[L_{g,t}|\cdot]) \\ RC_{g,t}^{f_{c,t}}: &\quad (E[L_{g,t}|f_{c,t}] - E[L_{g,t}|\cdot]) \end{aligned} \tag{16}$$

For example in our empirical results, we designate the causative sectors as the "Business" and the "Mortgages" for DBC and GFC crisis, respectively, and present them in Section 3.3.2.

3 Empirical Analysis

3.1 Data

For the empirical study, we use the quarterly loan sector level aggregated charge-off rates of the U.S. commercial banking system. These data are collected from the Federal Deposit Insurance Corporation(FDIC)[13] by loan type of six sectors including "Mortgages" (Real Estate Loans Secured by 1-4 Family Residential Properties.), "Business" (Commercial & Industrial Loans to U.S. Addressees.), "Credit Cards" (Credit cards.), "Individuals" (Other Loans to Individuals.), "Rest" (All Other Loans.), and "Lease" (Lease Financing Receivables.) during 1984:Q1~2019:Q3. These series include debt information from all companies and individual owners who are affiliated with the FDIC. These are employed by Ferrer et al. (2014), and similar to the previous studies using portfolio level default frequency (See Rosch and Scheule (2004); Lee and Yang (2019)). During the entire data period, we use the sub-period from 1990:Q1 to 2019:Q3 since all data of six research categories (sectors) are represented. The definition of obligors' default is the loan of 90 days delinquency[14] referred by Federal regulatory institutions. We calculated the loan sectors' charge-off rates as the amounts of total charge-offs dividing by the average outstanding loans each quarter. To obtain the annualized rates, we multiplied each quarterly charge-off rates by factor 4, because banks commonly measure credit risk over the one-year horizon. These series overlaps the three times economic downturn periods in the U.S. that have been defined by the National Bureau of Economic Research (NBER)[15]. These business cycles are CREC (the commercial real estate crisis during 1990:Q3~1991:Q1), DBC (the dotcom bubble crisis during 2001:Q1~2001:Q4), and GFC (the Great Financial Crisis during 2007:Q4~2009:Q2) that pointed to gray area in Fig. 1. However, in the empirical study in Section 3 presented only the analysis results 1991:Q2~2019:Q3, excluding the initial values u_{t-s} used for estimating the time-varying asset correlation model in Eq.(11).

Fig. 1 displays the charge-off rates of aggregated portfolios by sector. The charge-off rates were increasing stand out in almost all sectors around crisis windows. Each sector's charge-off increase with a slight time lag from the defined economic crisis period. This is a result of the definition of default by the 90 days delinquency and the time lag between the deterioration of the economic condition and obligors' financial states.

[Table 1 is here.]

Table 1 shows the descriptive statistics for 119 quarterly charge-off rates by sector. For exposure by sector, the Mortgages is the most at 46%, while the Lease has the lowest at around 3%. In particular, because the Mortgages and the Business account for around 70%

[13] The Loan Performance data is in https://www7.fdic.gov/idasp/advSearch_warp_download_all.asp?intTab=4

[14] This rule provides a good reason for using lagged macroeconomic variables for market expectation PD when build up the time-vary default threshold model in Section 2.2.1.

[15] https://www.nber.org/cycles.html

of the total loan market, the default rates in these two sectors are the most important for capturing the soundness of credit risk for the entire loan market. The Mortgages has the lowest default rate of 0.5% among all sectors. Also, the standard deviation of charge-off rate show a relatively low compared to the Business and the Creditcards sectors. In contrast, the Creditcards sector's charge-off rate displays the highest level and the biggest volatility among all sectors. As shown in Fig. 1, the volatility of the Creditcards is large during the crisis, this sector is most prominent for the default clustering. It should be noted that unlike other sectors, the Creditcards sector has the uniqueness as a high-level default rate that maintains a certain level for default even when the economy stabilizes.

And the charge-off distributions show the high left skewness in all sectors, it shows the fat-tail characteristics of the default data as compared to a normal distribution. This asymmetric unconditional loss distribution means that the default events concentrate in the economic downturn as called the default clustering. During the crisis periods in Fig. 1, each sector shows the default clustering, and the GFC around 2008 is the biggest impact for all sectors among the three crises. We should be noted that the dynamics of default rates in each sector differ from each economic crisis. For example, the Mortgages does not show significantly variability during other crises except the GFC while the Business and the Creditcards are seen as the most sensitive sectors in every crisis.

Many previous studies of credit risk fields have highlighted the economic adaptability for credit risk. For the corporate exposures, the significant macro economic covariates include the Real GDP growth, the S&P500 index return, the 3-month T-bill rate, and the spread between 10-year and 1-year Treasury note rates (see Koopman et al. (2011) and Duffie et al. (2009) etc.). The retail exposures relate to the GDP growth rate, the Unemployment rate, and the 3-month real interest rate (see Jiménez and Mencia (2009) as well as Lee and Poon (2014). etc.). For non-performing loans are surely explained by the GDP growth, the 30-year mortgage rate, the Consumer price index, the Industry production, the Prime loan rate, and the Housing price index (see Betz et al. (2020) and Ghosh (2017). etc.). In our time-varying default threshold model Eq. (9) is assumed to consist of lagged macroeconomic covariates by sector in order to identify the risk source $h_{g,t}$ for portfolio credit loss. And we use the expected loss $\Phi\left(h_{g,t}\right)$ from the macroeconomic information at time $t-\tau$. Based on previous research for the correlation between macroeconomic covariates and credit exposure, we consider for seasonally adjusted macro variables as the real GDP (GDP), the House Price Index (HPI), the Consumer price index (CPI), the Unemployment rate (UMEMP), and debt to income ratio (DTI) from Federal Reserve database. The market-based indicators are used the S&P500 index return (S&P500), the 1-year Treasury note rate (T_1Y), the 10-year (T_10Y), the interest rate spread between 10-year and 1-year Treasury note rates (Curvature), the 3-month T-bill rate (TB3MS), the TED Spread rate (TED) and the bank prime loan rate (Prime) in the Federal Reserve Economic Data. A total of 12 macroeconomic and market covariates are used in the three-step tests in section 2.2.1 for the time-varying default threshold (PD) model as raw data or differential terms.

3.2 Time-varying risk sources

Before examining the results of the time-varying models, we check the results of the static parameter model Eq. (7) for the portfolio by sector in Table 3 Panel C. This approach is similar to the current credit risk management standards for regulatory capital using the conservative risk parameters. The estimated expected loss $PD_g = \Phi(h_g)$ are similar level by sector to the mean of charge-off rates in Table 1. The estimated ρ_g for each sector portfolio represents by sector, respectively, which means the sensitivity to the common systematic risk factor is different for the loan type. These results show that the asset correlation rules by loan type in Basel are appropriate (see Committee et al. (2005) and BCBS (2019)) and the consistency of previous literature which suggest the vary asset correlation by loan sector (see Dietsch and Petey (2004a) and Bandyopadhyay et al. (2007). etc). Retail-related sectors with high default rates such as the Creditcards (1.9%) and the Individuals (1.6%) represent the relatively low asset correlation, while the Mortgages (10.4%) shows the high with a low default rate. These results support the Basel's decreasing function between default rate and asset correlation for the cross-section point of view. However, various empirical studies are examined the different results for the cross-sectional relations of the asset correlation and the default rates. They investigate that the relations can be positive or U shape according to credit grades or variety categories-size, industry, country, etc. (see Perli and Nayda (2004), , Düllmann and Scheule (2003), Dietsch and Petey (2004b), and Bandyopadhyay et al. (2007)etc.).

In the regulatory model based on the ASRF, the common systematic factor, which reflects the economic conditions, is integrated out in process of the driving unconditional loss distribution in Eq. (7). Thus, Basel recommends using sufficiently long-term data to reflect economic cycles include downturns when estimating risk components. Similarly, the default correlation among obligors within the portfolio expressed as asset correlation provides only the conservative criteria base on empirical results for G10 supervisors' data sets (see Committee et al. (2005)). But, these rules have an inevitable limitation in reflecting the change of economic condition point-in-time, even if the average value includes the extreme downturn. The risk sources within the portfolio are inconsistent and varies with changes in economic conditions.

Therefore, this paper estimates the dynamic portfolio loss models in Eq. (8) that applied the time-varying risk sources assumptions to default threshold h_g and asset correlation ρ_g which are the static parameters in unconditinal portfolio loss distribution in Eq. (7). The time-varying default threshold $h_{g,t}$ and time-varying asset correlation $\rho_{g,t}$ can identify the risk sources of realized portfolio loss.

3.2.1 Time-varying expected portfolio loss from macro

The first step is the time-varying modeling the portfolio expected loss $\Phi(h_{g,t})$ using solely lagged macroeconomic information $z_{k,t-\tau}$ in Section 2.2.1, we examine the cross-correlations between the time-lagged 12 macroeconomic covariates[16] and realized default thresholds $\Phi^{-1}(\ell_{g,t})$ by sector as shown in Table 2. We denote the $\tau - th$ cross correlations between realized default threshold of sector g and macro variable z_k are $Corr(\Phi^{-1}(\ell_{g,t}), z_{k,t-\tau})$.

[Table 2 is here.]

Each sector has slightly different covariates affected by the time difference, but the GDP growth (GDP diff), the Unemployment growth (Unemployment diff), the One-year interest rate change (T_1Y diff), and the Stock index (S&P500) show leaded cross-correlation across all sectors. Especially, the Debt-to-income ratio (DTI) and the TED Spread (TED) are significant explainable time leading covariates that the correlation of 4 quarters are strongest for the portfolio credit risk in all sectors except the Mortgages. In almost all sectors, the fourth-quarter time-lag covariates show a stronger correlation than others. In practice, these time-lagged covariates are important because they can be used to forecast increases in the sectors' credit risk even considering the timing of the announcement for economic variables and the charge-off rate data using 90 days delinquency. The Business and the Lease are found to be correlated with various economic covariates more than others.

Table 3 Panel A-1 shows the parameters β_k^z of the time-varying default threshold model in Eq. (10) applied the three steps process described in Section 2.2.1. Most of the covariates selected in the final models are among those with strong time-lags correlation in cross-correlation analysis in Table 2.

[Table 3 is here.]

The DTI ratio (time lag 4) associated with debt repayment capability is chosen as a significant covariate across all sectors except for the Mortgages. This sector's default rate is negatively affected by changes in the HPI index (time lag 4) and the GDP growth (time lag 4) which are not selected in other sectors. These results show that assets of the Mortgages, the Business, and personal-related sectors (the Individuals, the Credit cards) differ in macroeconomic covariates to consider depending on loan type. In this way, the time-varying expected loss can be estimated using only exogenous economic variables, and their estimates display the $PD_{g,t} = \Phi(h_{g,t})$ in Fig. 2. Moreover, the expected losses of unconditional loss distribution under static parameter assumption present the $PD_g = \Phi(h_g)$ in Fig. 2 and Table 3 Panel C.

[16] Total : 24 macroeconomic covariates (Raw data and similar differential terms-The GDP and the HPI are used to growth rate scale) in Section 3.1.

3.2.2 Time-varying asset correlation model from contagion

Table 3 Panel B shows the model parameters of $\rho_{g,t}$ in time-varying asset correlation in Eq. (11). The estimated parameters $\boldsymbol{\alpha_g} = (\alpha_{g,0}, \alpha_{g,1}, \alpha_{g,2})'$ are very significant over all sectors. The $\alpha_{g,1}$ means to the relationship between $\rho_{g,t}$ and $\rho_{g,t-1}$ for capturing the long-term memory of asset correlations over time. In other words, the positive significance sign of the bigger $\alpha_{g,1}$ means that asset correlation accelerates when it begins growing in economic downturns or upturns. Our results that the estimates of $\alpha_{g,1}$ are significant and positive in all sectors. This means that $\alpha_{g,1}$ is able to capture the auto-regressive characteristic for the default clustering similar to the frailty effect in previous studies. The persistence over the previous s lags time for short-term impact $\alpha_{g,2}$ are significant across all sectors. It reflects the volatility of the loss from dynamics for asset correlation within the portfolio and external macro shocks during s times. In summary, this model has combined the long-term trend effect $\alpha_{g,1}$ and the variability effect from short-term shocks $\alpha_{g,2}$ for asset correlation within the portfolio.

[Figure 2 is here.]

Fig 2 shows the estimates of time-varying asset correlation risk source $\rho_{g,t}$ at time t by sector with other estimated risk sources. The Creditcards (c), where the level of default and its volatility are higher than other sectors, presents the low-level asset correlation similar to the static model, although some increase around GFC and DBC. This sector's estimated parameters in Table 3 Panel B present the relatively larger $\alpha_{g,2}$ and smaller $\alpha_{g,1}$ than others. This means the Creditcards asset correlation is more affected by short-term shocks rather than long-term time dependence on them compared to others. Also, the mean of time-varying estimates presents the lowest level along with the Individuals similarly the results of static model ρ_g in Table 3. Panel C. These are called retail sectors, and the assets in the portfolio consist of a large number of borrowers and small-size exposure with relatively well-diversification. The default rate of the Creditcards $\ell_{Creditcards,t}$ is high due to a little (minor) delinquency for retail-related debt, but the asset correlation $\rho_{Creditcards,t}$ does not increase even in the economic crisis. High volatility for default rate during the economic downturn is due to also the base effect of the usual high default level even though normal economic conditions. The evidence for this phenomenon can be confirmed through the low level of relative standard deviation (RSD) in Table 1. These results remind the retail loan portfolios are a low systematic risk exposure resulting from macroeconomic changes rather than idiosyncratic factor effects.

The Mortgages (a), which has the lowest default rate, is a very important sector considering the about 46%[17] average market exposure weight and the variability in asset correlation during the economic crisis event periods. Asset correlation show a rapid increase since the 2008 GFC period and had continuously large volatility. And it was increasing sensitively to small economic shocks and reaching a peak in 2012. This means that the time gap between the

[17] The average weight of exposure by sector are described in Table 1.

recession and the default clustering of borrowers appears relatively slower than other loan sectors, and the shock of the recession continues for a considerable period of time and the default clustering is continuing. This shows that even if macroeconomic variables are recovered after the economic crisis, there is a time difference reflected in the stability of the credit portfolio. And the default clustering in this period can be underestimated by the expected default model by economic variables. In other words, the uncertainty that cannot be explained by observable covariates during the economic crisis has appeared, which can have the effect of underestimating the tail risk when evaluated by a static model. Based on the simultaneous or very small time difference between the increase in the default rate and the increase in asset correlation during the economic crisis, Basel's decreasing relation assumption is considered inappropriate to explain the default clustering during the economic crisis. Basel suggests the lower bounds for asset correlation function to compensate for its shortcomings. It is only a conservative approach for maintaining financial system stability, but there is a limit to identifying actual dynamics of risk sources that increase portfolio loss.

Particularly noteworthy is the different sector in which default clustering are concentrated in each economic crisis, and the dynamics of asset correlation are also different in each sector in each crisis. For example, looking at the Mortgages and the Business sectors can identify different movements of default rates and their risk sources by the crisis. In the case of the Mortgages, neither the default rate nor the asset correlation show large variability during the 2002 DBC but present sharply increasing and volatile during the 2008 GFC, indicating that it was the cause of the economic crisis. In contrast, in the case of the Business sector, default clustering is more prominent during the GFC, but asset correlation variability is greater during the DBC. The collapse of the Dotcom bubble seems to have caused default clustering as increasing the asset correlation (uncertainty) among the corporate obligors in the Business sector. These suggest that the causes of each economic crisis are different, and the increased risk in the origin sector that contributes may have caused the risk contagion to other sectors and lead to the overall economic crisis. Therefore, we simulate the contribution of each estimated risk source to the within portfolio loss as well as to the other sectors' one in the next Section 3.3.

Considering the effect of the two sectors as account for 70% of the total loan market, the default clustering occurring in different sectors during each crisis can be said to be a contagion or spillover effect throughout the entire economic system.

3.3 Contribution of risk sources

3.3.1 Risk source contribution within sector

Decomposing each source for the credit portfolio and measuring their contribution point-in-time could greatly utilize to ensure diversity in efficient hedge and portfolio optimization.

Thus, we present the contributions of estimated risk source point-in-time by sector in Fig 3 employed the Hoeffiding decomposition method using the conditional copula simulation in Section 2.3.2 and Appendix 3.

[Figure 3 is here.]

Fig. 3(a), 3(b), 3(c), 3(g), 3(h), and 3(i) show the partial expected losses within the portfolio given by each risk source. These were defined risk contribution $RC_{g,t}^{risk\,source}$ in Section 2.3.2. In other words, they present the portfolio base default rate $PD_{g,Base}$ unrelated to any given risk sources, the contribution $RC_{g,t}^{\Phi(h_{g,t})}$ of the expected default rate from the lagged macroeconomic variable, and the contribution $RC_{g,t}^{\rho_{g,t}}$ of estimated asset correlation within the portfolio g, respectively. And, Fig. 3(d), 3(e), 3(f), 3(j), 3(k), and 3(l) in the second row show the estimates of the time-varying risk sources $\Phi\left(h_{g,t}\right)$, $\rho_{g,t}$, and realized charge-off rate $\ell_{g,t}$, allowing each sector to compare estimates and contributions to portfolio losses.

The base level of default rates $PD_{g,Base}$ present the lowest the Mortgages (0.26%) in Fig. 3(a) and the highest the Creditcards (5.33%) in Fig. 3(c), similar to the descriptive statistics in Table1. This shows that the default rate per se is low (high) even if the Mortgage (the Creditcards) sector does not consider any risk sources. Furthermore, it can be seen that the most important factor in determining the portfolio losses in normal economic conditions is the base default rate of the portfolio per se. However, during the economic crisis, all sectors except retail-related sectors (the Creditcards (c) and the Individuals (d)) present a sharp increase in the contributions $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\rho_{g,t}}$ from risk sources, consequently exceed the contribution $PD_{g,Base}$ when default clustering occurs. What is the unusual point is that even in the economic crisis, retail-related portfolio losses are affected by more on the portfolio base default rate $PD_{g,Base}$ than risk sources such as macroeconomic conditon $\Phi\left(h_{g,t}\right)$ or asset correlation $\rho_{g,t}$. These sectors show that base default rates are the most important component in determining portfolio losses, although the contribution of different risk sources varies with economic fluctuations.

As shown the Mortgages (a) in Fig. 3, the contribution $RC_{Mortgages,t}^{\Phi\left(h_{Mortgages,t}\right)}$ for expected loss from macroeconomic covariates is rapidly increasing and then decreasing after as soon as the GFC. However, the realized default rate $\ell_{Mortgages,t}$ in Fig. 3(d) shows that the default clustering is continued without decreasing after the increase. At this time, the contribution $RC_{Mortgages,t}^{\rho_{Mortgages,t}}$ from asset correlation risk source in Fig. 3(a) has risen sharply, offsetting the decrease in the contribution $RC_{Mortgages,t}^{\Phi\left(h_{Mortgages,t}\right)}$ from macroeconomic covariates. As a result, the default clustering of the Mortgages sector in early the GFC origin from the expected default rate which is reflected in the macroeconomic shock. And after the macroeconomic covariates stabilized, the default rate did not decrease sharply because the asset correlation increasing.

In addition, the contributions $RC_{g,t}^{\Phi(h_{g,t})}$ from macroeconomic covariates have shown a sharp rise during the GFC in all sectors, as well as the Mortgages. These default clustering in most sectors are considered a predictable increasing for expected default rates by the recession based on observable macroeconomic information. The evidence, which such a predictable rising in default rates do not consider as uncertainty for other affected sectors, is that the contribution $RC_{g,t}^{\rho_{g,t}}$ of asset correlation in each sector does not present variability in GFC. Based on the results so far, the collapse of the Mortgages during the GFC is affecting the default clustering in other loan sectors by the channel of observable macroeconomic covariates. From a similar point of view, the Business sector's asset correlation contribution $RC_{Business,t}^{\rho_{Business,t}}$ and the macroeconomic contribution $RC_{Business,t}^{\Phi\left(h_{Business,t}\right)}$ in Fig. 3(b) exceed the $PD_{Business,Base}$ during the DBC crisis, but the asset correlation contributions $RC_{g,t}^{\rho_{g,t}}$ in other sectors do not variability significantly.

In summary, the base default rate $PD_{g,Base}$ within the portfolio is the most component for determining the portfolio loss in normal economic conditions, and in the crisis periods, the default rate increasing is able to express the expected default rate $\Phi\left(h_{g,t}\right)$ from observable macroeconomic covariates and the uncertainty as asset correlation $\rho_{g,t}$.

3.3.2 Risk source contribution across sector

We checked that each crisis originated from different sectors and each causative sector's default clustering contagious to the default of other sectors through channels such as macroeconomic covariates. In this section, we also simulate $RC_{g,t}^{\Phi(h_{c,t})}$ and $RC_{g,t}^{\rho_{c,t}}$ defind for the contribution from the risk sources of the causative sector c to the default rate $\ell_{g,t}$ of affected sectors. Based on results in Section 3.3.1 and prior knowledge, we choose the Business as the cause of the DBC and the Mortgages as GFC, then present the expected loss using the Hoeffiding decomposition by the across-sector conditional copula method defined in Section 3.3.2. More detail describes the simulation procedure in Appendix 3.

Fig. 4 and 5 represent the risk contributions $RC_{g,t}^{\Phi(h_{c,t})}$ and $RC_{g,t}^{\rho_{c,t}}$ for the realized loss $\ell_{g,t}$ of affected sectors g given the risk sources $\Phi\left(h_{cause,t}\right)$ and $\rho_{cause,t}$ of the Business and the Mortgages sector, respectively. The first row (a), (b), (e), (f), and (g) in each figure present the conditional expected loss $RC_{g,t}^{\rho_{c,t}}$ and $RC_{g,t}^{\rho_{g,t}}$ on portfolio g given the each estimated asset correlation risk sources $\rho_{cause,t}$ and $\rho_{g,t}$. And, the second row (c), (d), (h), (i), and (j) in each figure show the conditional expected loss $RC_{g,t}^{\Phi(h_{c,t})}$ and $RC_{g,t}^{\Phi(h_{g,t})}$ on portfolio g given the each estimated macroeconomic risk sources $\Phi\left(h_{cause,t}\right)$ and $\Phi\left(h_{g,t}\right)$. In this analysis, we confirm the different contagion channels by each crisis that different causative sectors.

Contagion effect from the Business sector during DBC The DBC economic crisis around 2002 was caused by the "Venture boom" of the ".com" corporates since the late 1990s. This collapse has become particularly concentrate on the defaults of corporate obligors, then we set the causative sector of the DBC economic crisis as the Business and investigate risk contagion effect to other sectors.

[Figure 5 is here.]

As shown Fig. 5(a), 5(b), 5(e), 5(f), and 5(g) during the DBC, the loss contributions $RC_{g,t}^{\rho_{Business,t}}$ in all most sectors from the Business sector's asset correlation present instantly significant increase except the Mortgages. Moreover, rather than the increase in asset correlation $RC_{g,t}^{\rho_{c,t}}$ within the portfolio, they represent that the increase in asset correlation $RC_{g,t}^{\rho_{Business,t}}$ from the Business contributes significantly. In particular, the Creditcards (c) is most affected sector, and changes are notable in retail-related sector such as the Individuals (f).

As shown Fig. 5(c), 5(d), 5(h), 5(i), and 5(j), the contributions $RC_{g,t}^{\Phi\left(h_{Business,t}\right)}$ and $RC_{g,t}^{\Phi(h_{g,t})}$ of expected losses implied in macroeconomic covariate increase with slight time lag in all sectors except the Mortgages. In particular, the Creditcards (d) is representing a significant increase compared to other sectors, because relative macroeconomic covariates for the Creditcards default rate consist of a subset in the Business sector's one shown as Table 3. Panel A-1. Moreover, in the case of the HPI and the Unemployment covariates, the time lags are also greater the Creditcards than the Business resulting in a more time difference and an increase in the contribution to the loss.

Contagion effect from the Mortgages sector during GFC The Mortgages sector collapse, which known as the cause of the GFC crisis, contributes to the loss of other sectors as shown in Fig. 4. As shown (a), (b), (e), (f), and (g), the risk contribution $RC_{g,t}^{\rho_{Mortgages,t}}$ of asset correlation from the Mortgages is stable in this period. This means that the rapidly increasing asset correlation $\rho_{Mortgages,t}$ of the Mortgages during the GFC crisis does not directly contagious to the risk of other sectors by the channel as asset correlation.

[Figure 4 is here.]

As shown Fig. 4(c), 4(d), 4(h), 4(i), and 4(j), the contribution of expected default rates from macroeconomic covariates also present as greater from the Mortgages $RC_{g,t}^{\Phi\left(h_{Mortgages,t}\right)}$ than from within portfolio $RC_{g,t}^{\Phi(h_{g,t})}$. However, the contributions $RC_{g,t}^{\Phi(h_{g,t})}$ from macroeconomic covariates in each sector have been increasing rapidly. In particular, the Business, the Creditcards, and the Rest sectors are presenting remarkable variabilities. These are due to results in Table 3 Panel A-1 , which the relevant macroeconomic covariates differ to the Mortgages and other sectors. This implies that although the losses of affected sectors are not relevant to the covariates of the Mortgages sector, their covariates are exacerbated by the collapse of the Mortgages then increasing macroeconomic risk source contribution $RC_{g,t}^{\Phi(h_{g,t})}$ within the portfolio.

Eventually, the default clustering of the Mortgages during the GFC did not directly contagious to other portfolios, but has a negative impact on the economy. Thus we can identify the macroeconomic covariates as risk contagion channel. This means that the collapse in the Mortgages during the GFC crisis roles as a systemic risk event for the entire financial system similar as in Lee and Yang (2019). A noteworthy point is that the deterioration of macroeconomic conditions has led to the default clustering of other sectors, but the asset correlation in each sector has not increased significantly. This can be reaffirmed that the predictable risk is not considered the uncertainty, and do not present the additional increase of asset correlation among obligors.

3.4 Compare the frailty effects

To check the adequacy of our time-varying risk sources model, which is decomposed the frailty effect into asset correlation dynamics and pure frailty effect, we examine the residual frailty effects with employ various assumptions to risk sources of portfolio loss. Fig. 6 shows the contribution $RC_{g,t}^{f_{g,t}}$ for within portfolio loss from residual frailty effects in Section 2.3.1.

[Figure 6 is here.]

The Fig. 6 shows that the dynamics of residual frailty effects based on a static parameter assumption model Eq. (4) are similar to the default clustering by each sector. However, the residual frailty effect based on the time-varying risk sources model Eq.(13) show the more random dynamics that eliminate the effects of many default clustering although some volatility remains during the economic crisis.

Our time-varying methods are intuitive and can be represented by the dynamics of asset correlation on the basis of the regulatory framework without expansion of systematic factors, for the default clustering explanation effects of frailty presented in previous studies (see Duffie et al. (2009); Koopman et al. (2012); Lee and Poon (2014). etc.). Moreover, we can evaluate the time-varying parameter assumption model better represents the dynamics of the portfolio realized loss $\ell_{g,t}$ by at the Log-likelihood values of each model presented in the last row of Panel B and Panel C in Table 3. Although not reported, the log-likelihood ratio tests for the static vs the time-varying model have shown significant results at the confidence level of 99.9% in all sectors.

4 Conclusion

In managing and regulating the credit risk of loan portfolios, measurement and rational estimation of their risk sources are very important for both regulators and financial institutions. Our paper has presented simple and easy methods for time-varying credit portfolio loss distribution and for their sources decomposing base on the ASRF model under the LHP assumption, which is the basis for the standard credit risk management process. And, we propose a procedure for estimating the contribution of risk sources and for managing them when the economic conditions are given at each point in time. Furthermore, we investigate that the frailty effect, which is considered an latent factor to explain the exceed default clustering, can be divided into time-varying asset correlation dynamics and frailty effect. Our proposed methods are able to reduce the model risk for Basel's one-factor model under static assumptions by monitoring the default clustering occurring during the crisis into observable and controllable risk sources. Our methodology could identify not only the cross-sectional diversity (eg. sector, region, credit grade etc.), but also the dynamics of loan assets comovement that comprises the portfolio held by financial institutions.

With empirical application, we show that the contributions of time-varying risk sources for portfolio loss are different in normal or crisis economic conditions. During the normal economic conditions, the base default rate level is the most important source to determine the portfolio loss than other risk sources. But, the effects of macroeconomic and asset correlation are increasing during the crisis resulting in the default clustering within portfolio or the financial systemic risk. In particular, the dynamics of risk sources are different in the causative sector and affected sector. The origin sectors of each crisis show the rapid increase in asset correlation because of the uncertainty from the exceeding default clustering over the expected default rate inherent in macroeconomic variables. And, there is the contagion effect to other sectors through the channel for macroeconomic variables or asset correlation according to type of each crisis. In the case of affected sectors, the contagion channels are found to be most affected by macroeconomic variables, followed by the effect of increasing asset correlation across the portfolio, and by the frailty effect.

Our proposed methodology could be a practically useful tool for not only regulatory authorities but also financial institutions to grasp the dynamics of the actual risk sources on the portfolio depending on economic conditions without additional data under the current credit risk management system. This approach is expected to contribute to securing the stability of the portfolio level and the entire financial system by the fat tail risk responding to by sources when the economic downturn.

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Tables

Table 1: Descriptive statistics for charge-off rates by sector in annualized

This table shows the descriptive statistics for charge-off rate by sector in 1990:Q1~2019:Q3. Charge-off historical data in FDIC covers since 1984 :Q1 all sectors except the Mortgages, but this paper is used sub time horizon(1990:Q1~) data that all sectors are covered. Because the Mortgages sector is important that exposure weight is 46% on average all over time.

	Mortgages	Business	Creditcards	Individuals	Rest	Lease
Weight*	0.46	0.23	0.09	0.13	0.06	0.03
N	119	119	119	119	119	119
Mean	0.0045	0.0108	0.0546	0.0159	0.0059	0.0059
Std	0.0059	0.0072	0.0177	0.0058	0.0056	0.0036
Skew	1.8708	1.2294	2.1781	1.7169	2.6095	1.2235
Kurt	2.2970	0.7219	6.3780	3.1119	7.4004	1.0525
Min	0.0005	0.0033	0.0347	0.0086	0.0016	0.0017
Q1	0.0012	0.0053	0.0437	0.0125	0.0025	0.0033
Med	0.0017	0.0078	0.0504	0.0144	0.0040	0.0046
Q3	0.0040	0.0150	0.0604	0.0178	0.0066	0.0085
Max	0.0254	0.0332	0.1444	0.0362	0.0311	0.0177
RSD**	1.3111	0.6667	0.3242	0.3648	0.9492	0.6102

* $Weight = average(sector's\, exposure/total\, exposure)$

** The RSD is relative standard deviation.($RSD = Std/Mean$)

Table 2: Cross correlation for expected PD model

This table presents the effective pearson correlation of realized default threshold $\Phi^{-1}(\ell_{g,t})$ and time-lagged macroeconomic covariates $z_{k,t-\tau}$ that satisfied the economic perspective expected sign and statistical significance level at 5%. The reporting numbers denote the leading time τ on quarterly for macroeconomic coavriates. And the sequence of numbers means an order by the absolute value of the pearson correlation. For example, the case of the GDP for the Mortgages is effective time-lagged correlation coefficients are only in the differential terms. And the sequence of 4 3 2 1 means that an absolute value of 4 quarters lagged correlation coefficient is bigger than 3 quarters it. That means $Corr(\Phi^{-1}(\ell_{g,t}), GDP_{t-4}/GDP_{t-5}) > Corr(\Phi^{-1}(\ell_{g,t}), GDP_{t-3}/GDP_{t-4})$. In addition, we have tested not only the time-lagged correlations but also the backwardness of economic variables. However, we are interested in the time-lagged correlation of macro variables, so we do not report that.

		Mortgages	Business	Creditcards	Individuals	Rest	Lease
GDP	raw						
	diff	4 3 2 1	3 4 1	4 3 2 1	3 1 4 2	3 2 4 1	3 2 4 1
HPI	raw						
	diff	4 3 2 1	2 3 4 1	3 4 2 1			3 2
CPI	raw		1 2 3 4				1 2 3 4
	diff						
Unemployment	raw						
	diff	4 3 2 1	3 4 2 1	4 3 2 1	2 3 1 4	2 3 1 4	4 3 2 1
DTI	raw		4 3 2 1	4 3 2 1	4 3 2 1	4 3 2 1	4 3 2 1
	diff						
S&P500	raw	4	4 3 1 2	4 3	4 1 2 3	3 4 1 2	4 2 3 1
	diff						
T_1Y	raw		4				4 3
	diff	3 4 2 1	3 4 2 1	4 3 2 1	1 4 3 2	3 2 1 4	4 3 2 1
T_10Y	raw					4	
	diff						
Curvature	raw						
	diff		2 3 4 1			2 3 1 4	4 2 3
TB3MS	raw						
	diff	4 3 2 1					
TED	raw		4 3 2	4 3 2	4 3 2 1	4 3 2 1	4 3 2
	diff		3 1				
Prime	raw		4				4 3
	diff	4 3 2 1	3 2 4 1	4 3 1 2			4 3 2 1

Table 3: Time-varying model prameters

Panel A : This table presents the estimated parameters for time-varying default thresholds $h_{g,t}$ in Eq. (9) and time-varying asset correlation $\rho_{g,t}$ in Eq.(11), and the summary descriptive statistics for $\rho_{g,t}$ and $h_{g,t}$ point-in-time. One, two, and three asterisks indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively. The $PD_{g,t}$ indicate the expected portfolio loss $PD_{g,t} = \Phi(h_{g,t})$, and Δ means the differential terms.
Panel C: This table shows the estimeted asset correlation ρ_g and default threshold h_g based on static parameters assumption model in Eq. (7). The PD_g indicate the unconditinal expected default rate $PD_g = \Phi(h_g)$.

Panel A-1 : Time-varying default threshold model $h_{g,t}$.

Variable name	Mortgages	Business	Creditcards	Individuals	Rest	Lease
constant	-2.502 ***	-2.507 ***	-2.495 ***	-3.023 ***	-3.874 ***	-4.087 ***
CPI_{t-1}		-0.005 ***				
ΔDTI_{t-4}		0.062 ***	0.079 ***	0.077 ***	0.107 ***	0.140 ***
ΔGDP_{t-3}		-6.518 ***		-2.236 **		-6.345 ***
ΔGDP_{t-4}	-14.635 ***					
ΔHPI_{t-2}		-3.448 ***				
ΔHPI_{t-3}			-1.386 **			
ΔHPI_{t-4}	-18.266 ***					
$\Delta Prime_{t-3}$		-0.122 **				
$\Delta Prime_{t-4}$						-0.072 ***
T_1Y_{t-4}		-0.025 **				
TED_{t-4}					0.168 ***	
$\Delta Unemploy_{t-2}$					0.268 ***	
$\Delta Unemploy_{t-3}$		0.202 ***				
$\Delta Unemploy_{t-4}$			0.099 ***			0.078 **
$Mean(PD_t)$	0.0042	0.0098	0.0537	0.0156	0.0057	0.0061
$Std(PD_t)$	0.0051	0.0065	0.0118	0.0033	0.0048	0.0035
$corr(PD_t, \ell_t)$	0.7681	0.8376	0.8170	0.8279	0.8890	0.8012

Panel B : Time-varying asset correlation model $\rho_{g,t}$.

Variable name	Mortgages	Business	Creditcards	Individuals	Rest	Lease
α_0	-0.401	-0.536 ***	-0.623 ***	-0.631 ***	-0.463 ***	-0.556 ***
α_1	1.359 ***	2.572 ***	0.835 ***	4.631 ***	1.056 ***	5.146 ***
α_2	0.019 ***	0.045 ***	0.051 ***	0.055 ***	0.025 **	0.021 ***
$Mean(\rho_t)$	0.0412	0.0132	0.0057	0.0053	0.0156	0.0118
$Std(\rho_t)$	0.0218	0.0114	0.0120	0.0061	0.0064	0.0110
s	2	1	3	2	5	5
Log-likelihood	572.00	521.30	395.86	524.59	571.87	584.28

Panel C : Static model $\rho_g$and h_g.

Variable name	Mortgages	Business	Creditcards	Individuals	Rest	Lease
PD	0.0042	0.0104	0.0550	0.0160	0.0056	0.0058
ρ	0.1036	0.0493	0.0188	0.0163	0.0546	0.0367
Log-likelihood	513.07	435.05	317.69	444.23	496.62	509.07

Table 4: Various model prameters

Panel A : This table presents the estimated parameters for risk sources that asset correlation ρ_g based on static assumption, and default threshold h_g based on time-varying assumption. One, two, and three asterisks indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively. The $PD_{g,t}$ indicate the expected portfolio loss $PD_{g,t} = \Phi(h_{g,t})$, and Δ means the differential terms.
Panel B: This table shows the estimeted asset correlation $\rho_{g,t}$ based on time-varying assumption, and default threshold h_g based on static assumption. The PD_g indicate the unconditinal expected default rate $PD_g = \Phi(h_g)$.

Panel A : Time-varying default threshold model $h_{g,t}$ under static asset correlation.

Variable name	Mortgages	Business	Creditcards	Individuals	Rest	Lease
constant	-2.451 ***	-2.675 ***	-2.234 ***	-3.177 ***	-3.969 ***	-3.650 ***
CPI_{t-1}		-0.005 ***				
ΔDTI_{t-4}		0.079 ***	0.058 ***	0.094 ***	0.117 ***	0.098 ***
ΔGDP_{t-3}		-6.157 ***		-6.554 ***		-3.907 ***
ΔGDP_{t-4}	-16.139 ***					
ΔHPI_{t-2}		-3.001 ***				
ΔHPI_{t-3}			-2.372 **			
ΔHPI_{t-4}	-19.222 ***					
$\Delta Prime_{t-3}$		-0.099 **				
$\Delta Prime_{t-4}$						-0.112 ***
T_1Y_{t-4}		-0.027 ***				
TED_{t-4}					0.116 ***	
$\Delta Unemploy_{t-2}$					0.267 ***	
$\Delta Unemploy_{t-3}$		0.198 ***				
$\Delta Unemploy_{t-4}$			0.230 ***			0.149 **
$Mean(PD_t)$	0.0048	0.0103	0.0556	0.0161	0.0055	0.0057
$Std(PD_t)$	0.0061	0.0066	0.0153	0.0047	0.0042	0.0031
$corr(PD_t, \ell_t)$	0.7638	0.8377	0.8756	0.8511	0.8842	0.8262
ρ(static)	0.0392	0.0138	0.0052	0.0047	0.0158	0.0110
Log-likelihood	551.66	494.99	377.79	497.66	552.55	561.29

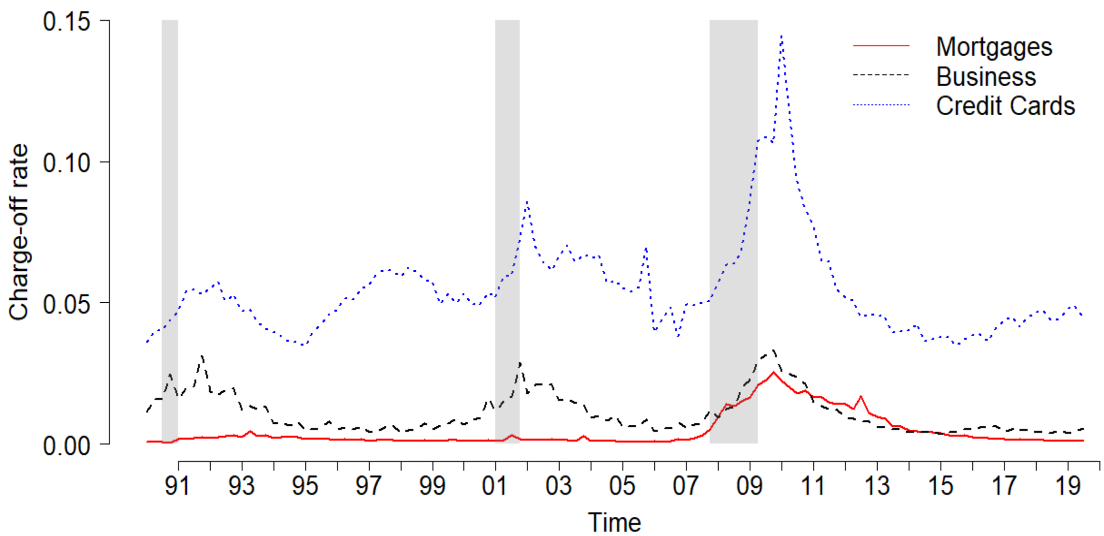
Panel B : Time-varying asset correlation model $\rho_{g,t}$ under static default threshold.

Variable name	Mortgages	Business	Creditcards	Individuals	Rest	Lease
α_0	-0.397***	-0.404	-0.653 ***	-0.054 ***	-0.359	-0.374
α_1	0.642 ***	0.000	0.000	2.170 ***	0.000	0.000
α_2	0.023 ***	0.047 ***	0.149	0.046 ***	0.046	0.022
$Mean(\rho_t)$	0.080	0.0445	0.0180	0.0144	0.0491	0.0353
$Std(\rho_t)$	0.121	0.0671	0.0331	0.0201	0.0332	0.0292
s	5	4	4	2	2	4
PD(static)	0.0020	0.0087	0.0502	0.0152	0.0055	0.0055
Log-likelihood	530.66	443.24	355.92	470.49	507.23	511.64

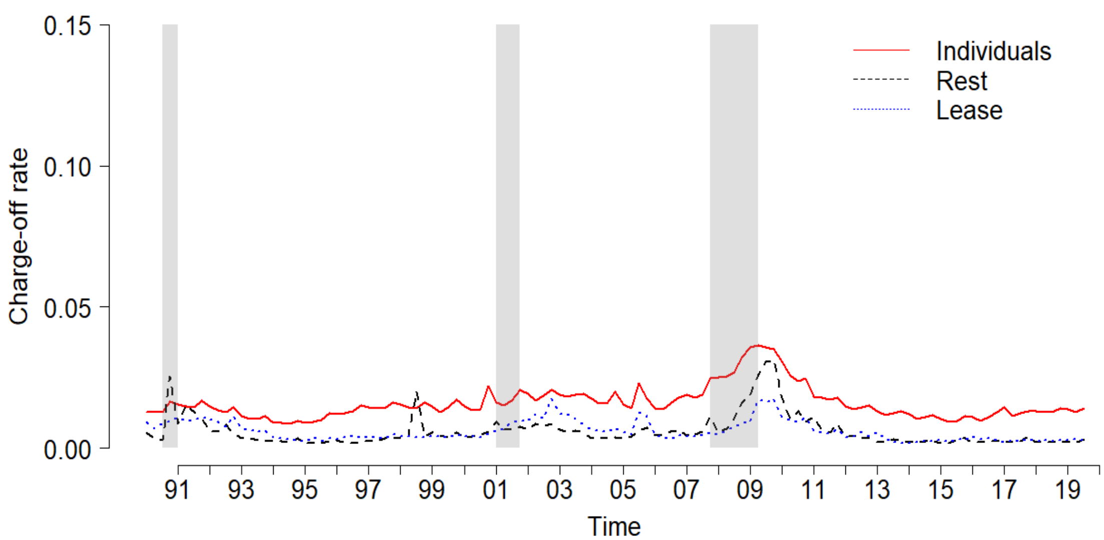
Figures

Figure 1: Annualized charge-off rates

This figure presents the historical annualized charge-off rates by loan sector during 1990:1Q~2019:3Q. The gray bars show the U.S. business cycle contraction periods: the Commercial Real Estate Crisis from 1990:Q3 to 1991:Q1 (CREC), the Dotcom Bubble Crisis from 2001:Q1 to 2001:Q4 (DBC) and the Great Financial Crisis from 2007:Q4 to 2009:Q2 (GFC) defined by the National Bureau of Economic Research (NBER).



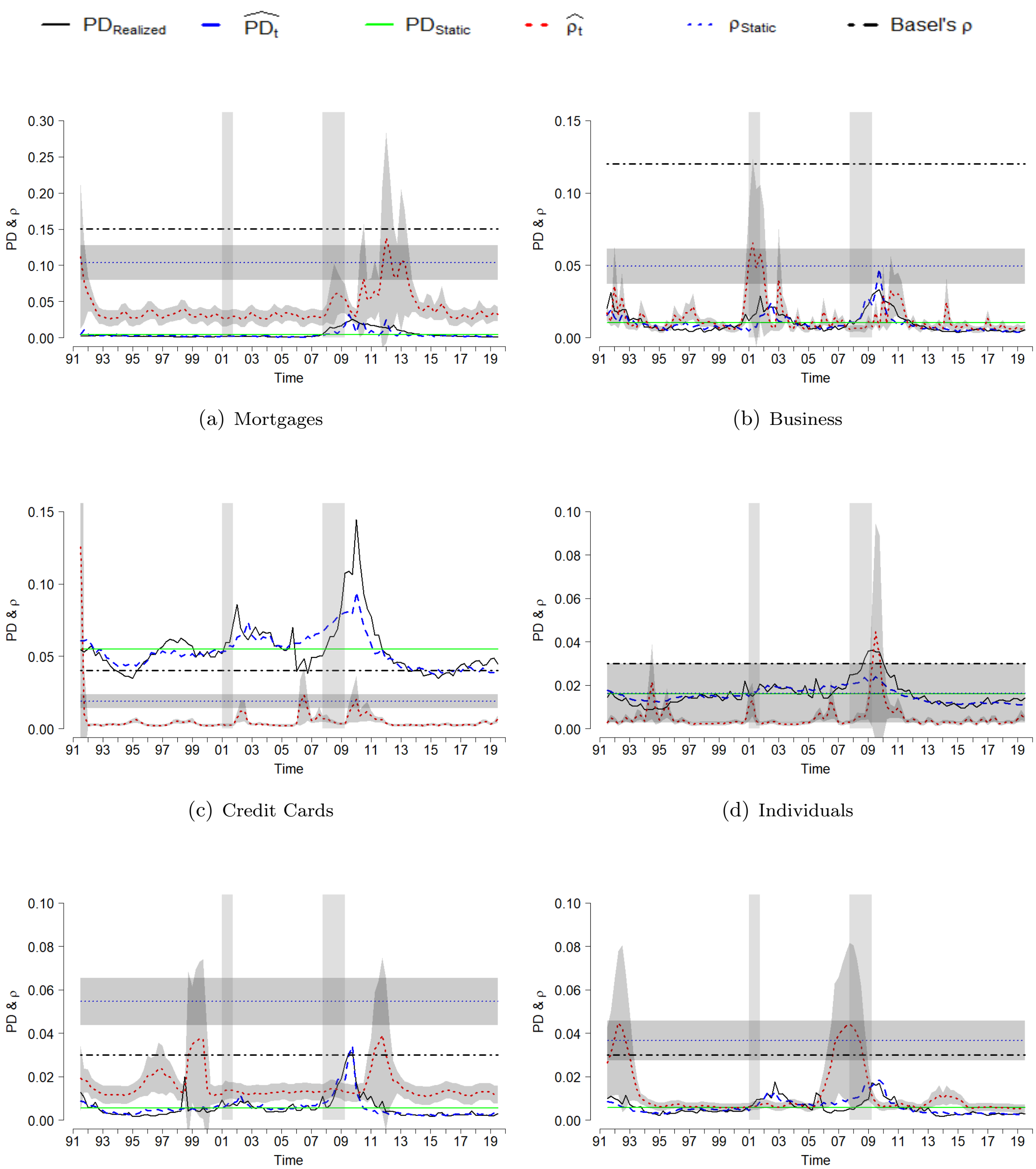
(a) Mortgages, Business and Credit Cards



(b) Individual, Rest and Lease

Figure 2: Time-varying risk sources and sector charge-off

This figure compares the estimated time-varying risk sources ($\rho_{g,t}$, $PD_{g,t} = \Phi(h_{g,t})$) and the static risk sources (ρ_g,$PD_g = \Phi(h_g)$) with historial annualized charge-off rates by sector. The axis range is marked differently for each sector to show information more efficiently. (e.g. the Mortgages:0~0.3, the Business and Creditcards:0~0.15, others:0~0.1). The gray band around the two asset correlation estimates ($\rho_{g,t}$, ρ_g) are the 95% confidence interval using the delta method. Basel's criterias show constant value or lower bound of mapping exposure class in Appendix 1.



(a) Mortgages

(b) Business

(c) Credit Cards

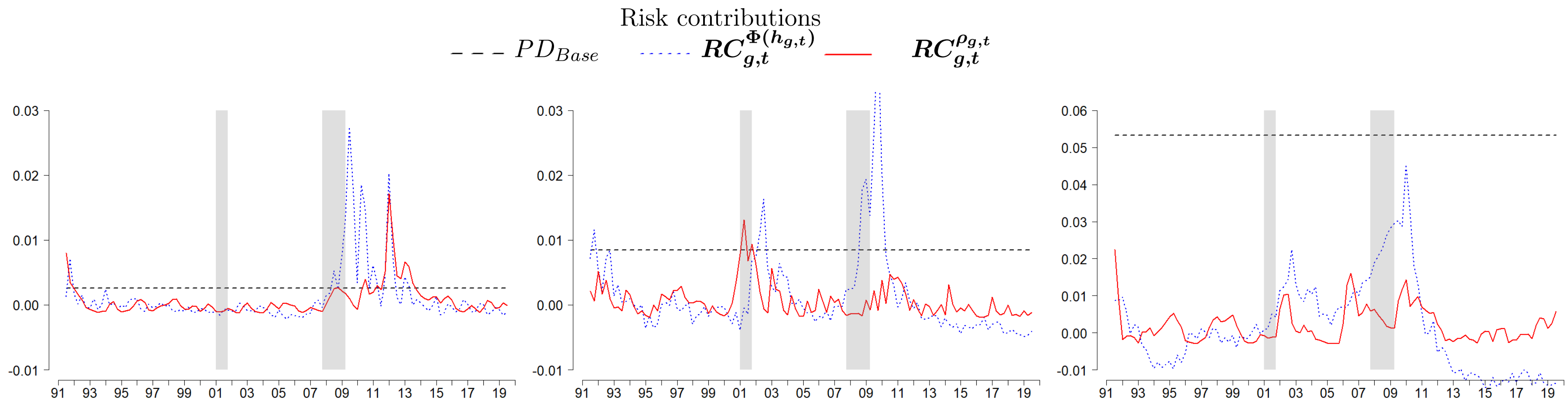
(d) Individuals

(e) Rest

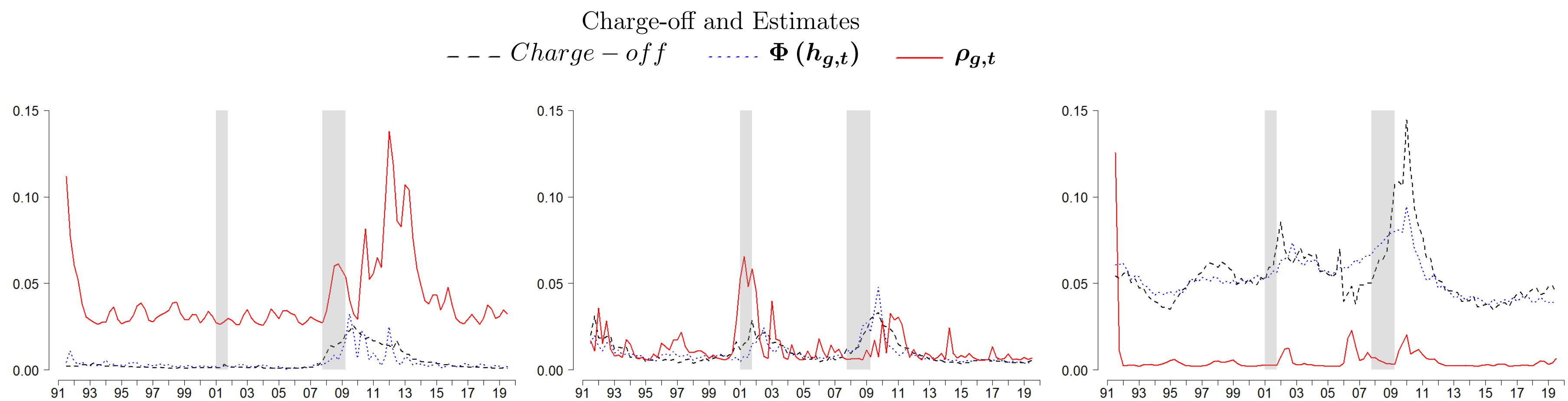
(f) Lease

Figure 3: Risk sources contribution within sector

This figure shows the estimated risk sources and their risk contributions for portfolio loss by sector. These are represented in pair pictures by sector to compare the dynamics of risk sources and their contributions by time. The above line presents the base default rate level $PD_{g,Base}$, and the risk contribution $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\rho_{g,t}}$ given time-varying risk sources $\Phi(h_{g,t})$ and $\rho_{g,t}$, respectively in Eq. (15). And, the below line presents the realized charge-off rate $\ell_{g,t}$ and the estimated time-varying risk sources $\Phi(h_{g,t})$ and $\rho_{g,t}$. This figure contains sectors of the Mortgages, the Business, and the Creditcards.



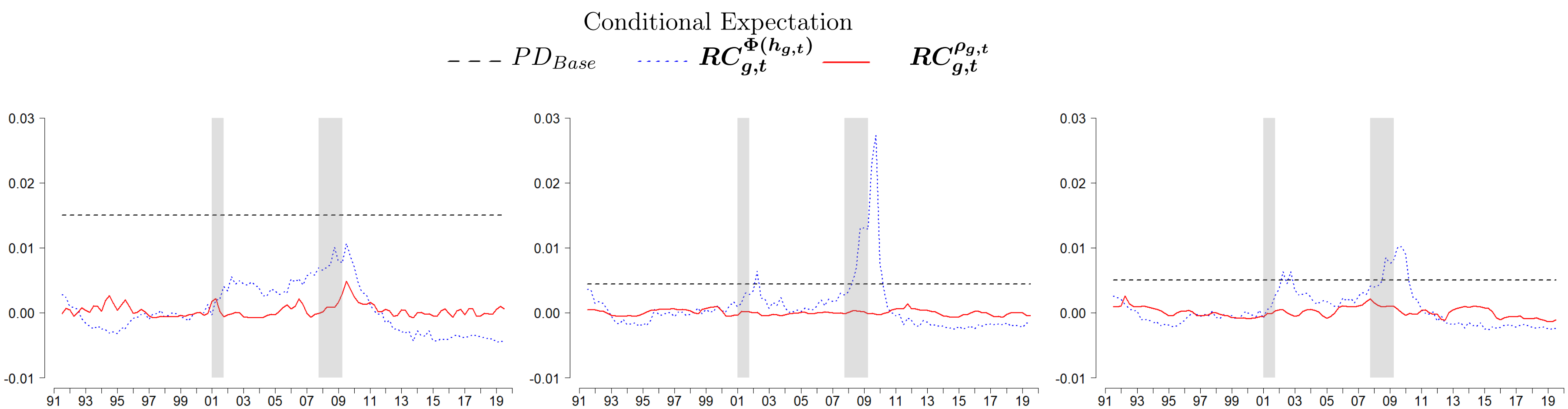
(a) Mortgages (b) Business (c) Credit cards



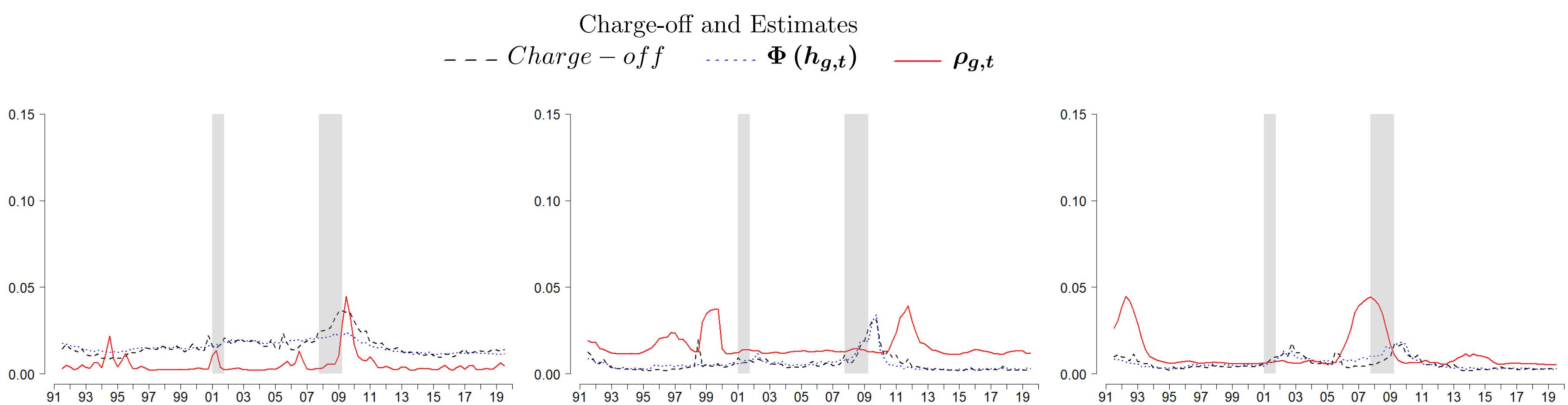
(d) Mortgages (e) Business (f) Credit cards

Figure 3-2: Risk sources contribution within sector(Continue)

This figure shows the estimated risk sources and their risk contributions for portfolio loss by sector. These are represented in pair pictures by sector to compare the dynamics of risk sources and their contributions by time. The above line presents the base default rate level $PD_{g,Base}$, and the risk contribution $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\rho_{g,t}}$ given time-varying risk sources $\Phi(h_{g,t})$ and $\rho_{g,t}$, respectively in Eq. (15). And, the below line presents the realized charge-off rate $\ell_{g,t}$ and the estimated time-varying risk sources $\Phi(h_{g,t})$ and $\rho_{g,t}$. This figure contains sectors of the Individuals, the Rest, and the Lease.



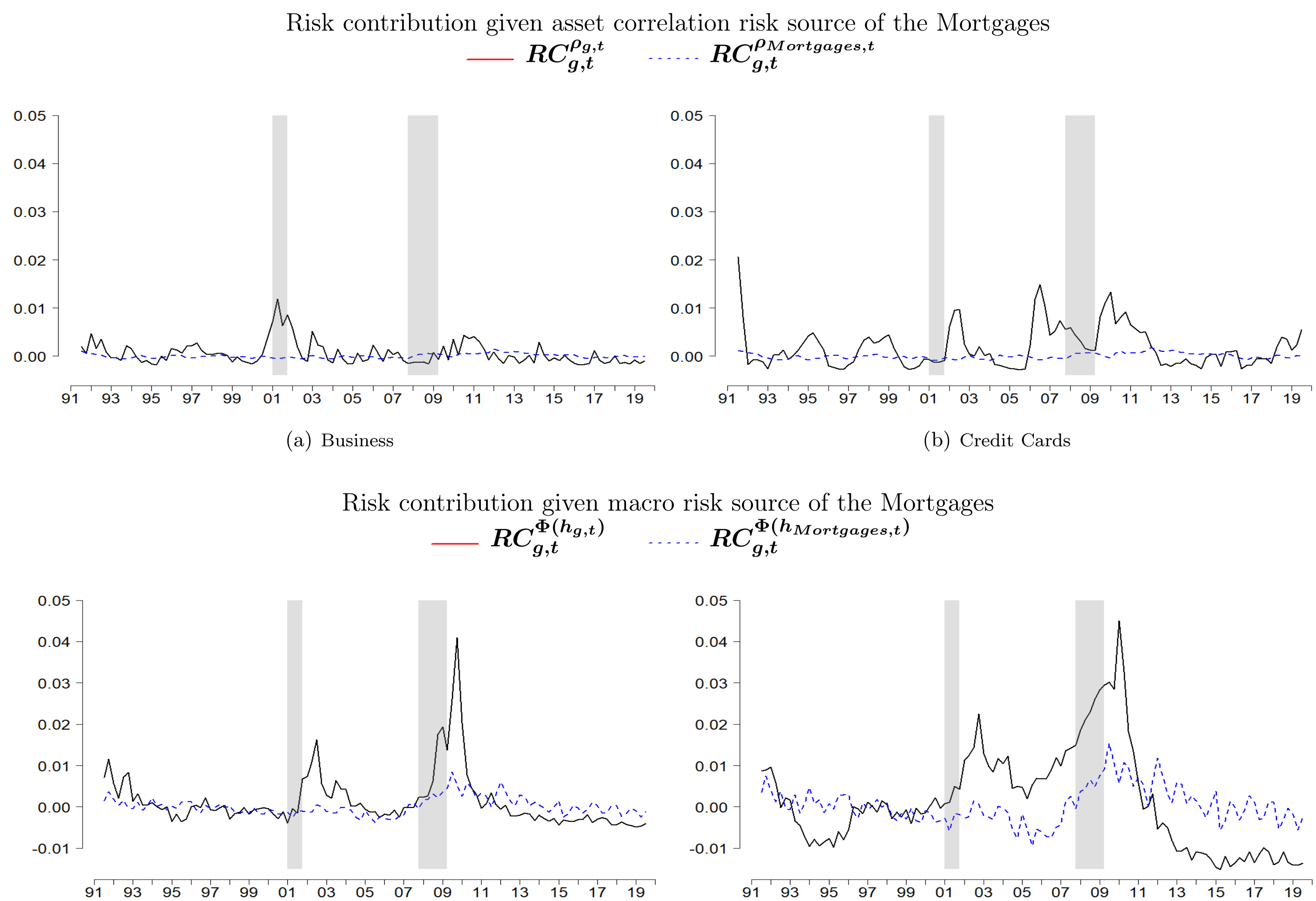
(g) Individuals (h) Rest (i) Lease



(j) Individuals (k) Rest (l) Lease

Figure 4: Risk sources contribution across sector from the Mortgages sector.

This figure presents the estimated risk sources and their risk contributions for portfolio loss of affected sector g (the Business and the Creditcards) from the causative sector (the Mortgages) by sector. These are represented in pair pictures by sector to compare the effect of macroeconomic and asset correlation by time. The above line presents the risk contributions $RC_{g,t}^{\rho_{g,t}}$ and $RC_{g,t}^{\rho_{Mortgages,t}}$ for sector g given asset correlation within sector $\rho_{g,t}$ or causative sector $\rho_{Mortgages,t}$, respectively in Eq. (16). And, the below line shows the risk contributions $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\Phi(h_{Mortgages,t})}$ for sector g given estimated default thresholds within sector $\Phi\left(h_{g,t}\right)$ or causative sector $\Phi\left(h_{Motgages,t}\right)$, respectively in Eq. (16).



(a) Business

(b) Credit Cards

(c) Business

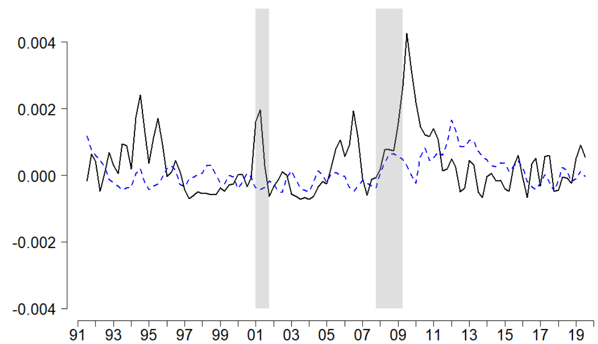
(d) Credit Cards

Figure 4: Risk sources contribution from the Mortgages sector(Continue)

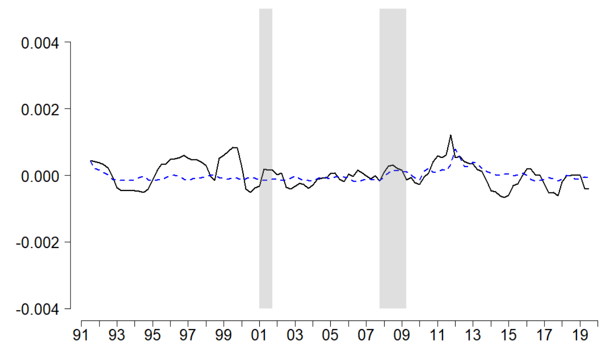
This figure presents the estimated risk sources and their risk contributions for portfolio loss of affected sector g (the Individuals, the Rest, and the Lease) from the causative sector (the Mortgages) by sector. These are represented in pair pictures by sector to compare the effect of macroeconomic and asset correlation by time. The above line presents the risk contributions $RC_{g,t}^{\rho_{g,t}}$ and $RC_{g,t}^{\rho_{Mortgages,t}}$ for sector g given asset correlation within sector $\rho_{g,t}$ or causative sector $\rho_{Mortgages,t}$, respectively in Eq. (16). And, the below line shows the risk contributions $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\Phi(h_{Mortgages,t})}$ for sector g given estimated default thresholds within sector $\Phi\left(h_{g,t}\right)$ or causative sector $\Phi\left(h_{Motgages,t}\right)$, respectively in Eq. (16).

Risk contribution given asset correlation risk source in the Mortgages

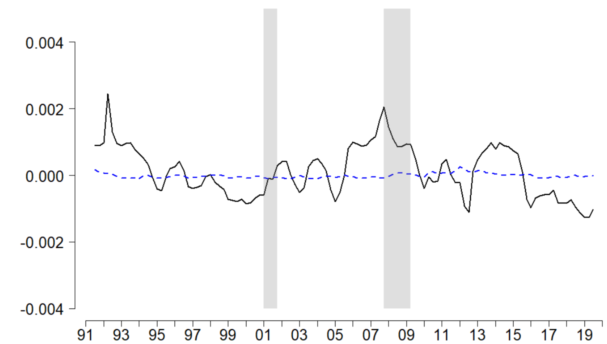
$RC_{g,t}^{\rho_{g,t}}$ $RC_{g,t}^{\rho_{Mortgages,t}}$



(e) Individuals



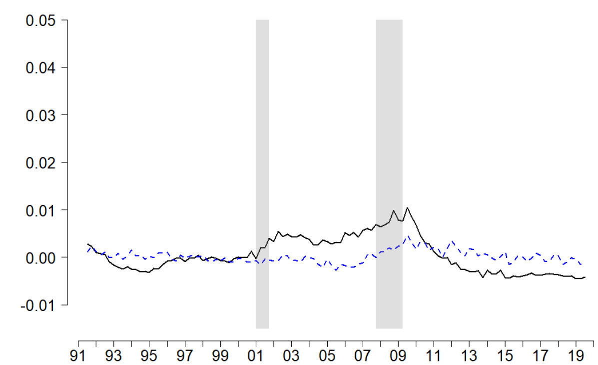
(f) Rest



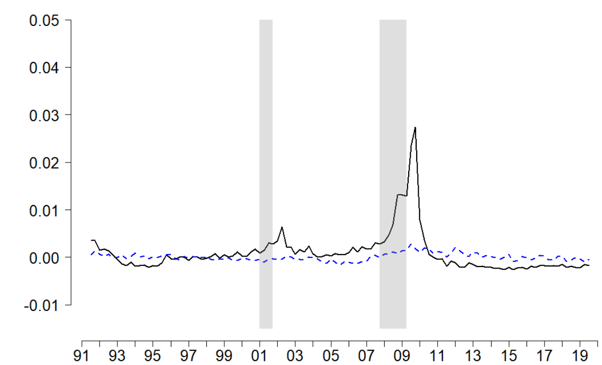
(g) Lease

Risk contribution given macro risk source in the Mortgages

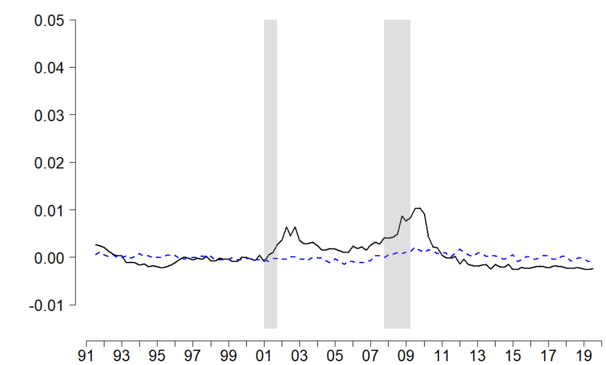
$RC_{g,t}^{\Phi(h_{g,t})}$ $RC_{g,t}^{\Phi(h_{Mortgages,t})}$



(h) Individuals



(i) Rest



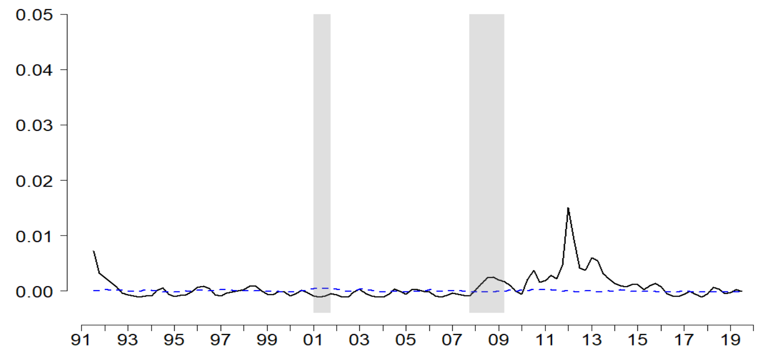
(j) Lease

Figure 5: Risk source contribution across sector from the Business sector.

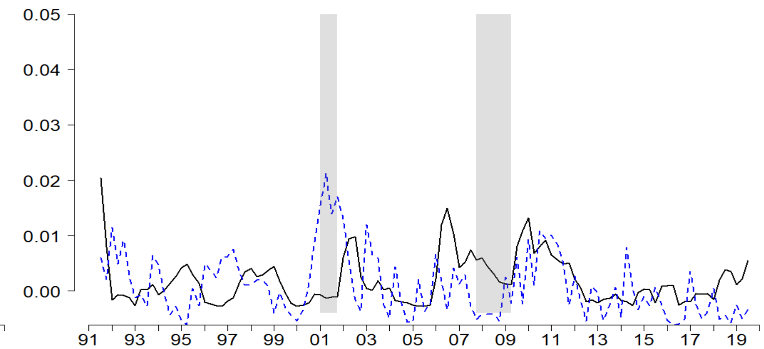
This figure presents the estimated risk sources and their risk contributions for portfolio loss of affected sector g (the Mortgages, the Creditcards) from the causative sector (the Business) by sector. These are represented in pair pictures by sector to compare the effect of macroeconomic and asset correlation by time. The above line presents the risk contributions $RC_{g,t}^{\rho_{g,t}}$ and $RC_{g,t}^{\rho_{Business,t}}$ for sector g given asset correlation within sector $\rho_{g,t}$ or causative sector $\rho_{Business,t}$, respectively in Eq. 16. And, the below line shows the risk contributions $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\Phi(h_{Business,t})}$ for sector g given estimated default thresholds within sector $\Phi(h_{g,t})$ or causative sector $\Phi(h_{Business,t})$, respectively in Eq. 16.

Risk contribution given asset correlation risk source in the Business

$RC_{g,t}^{\rho_{g,t}}$ $RC_{g,t}^{\rho_{Business,t}}$



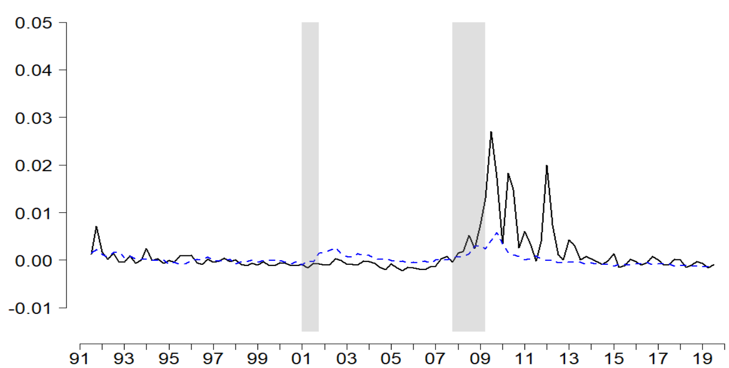
(a) Mortgages



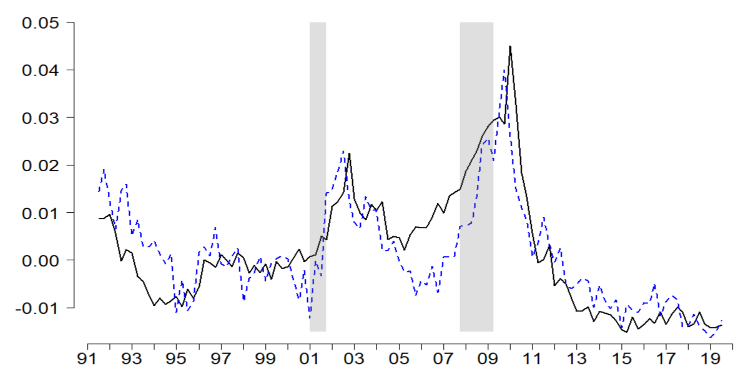
(b) Credit Cards

Risk contribution given macro risk source in the Business

$RC_{g,t}^{\Phi(h_{g,t})}$ $RC_{g,t}^{\Phi(h_{Business,t})}$



(c) Mortgages

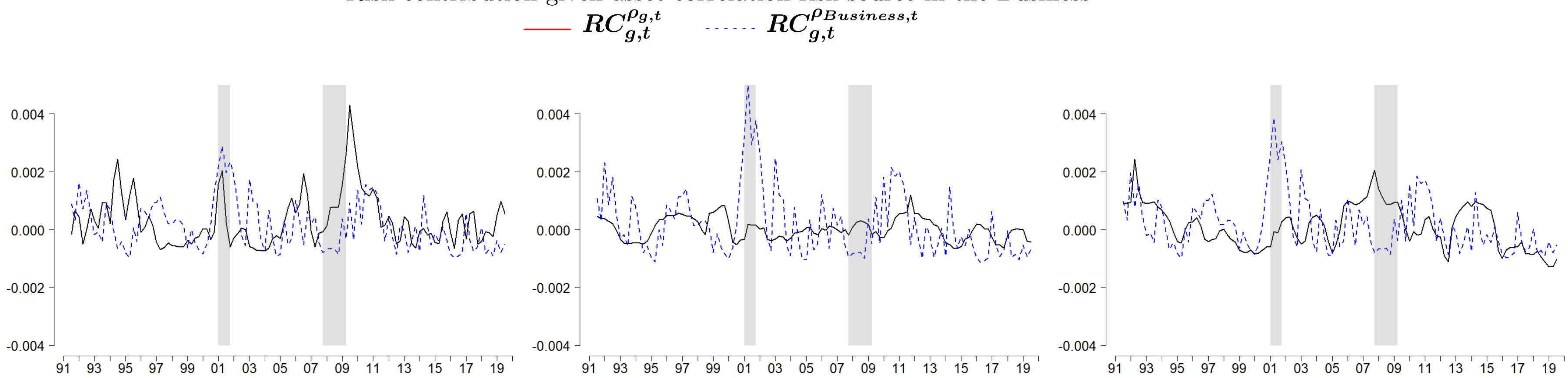


(d) Credit Cards

Figure 5: Risk sources contribution from the Business sector(Continue)

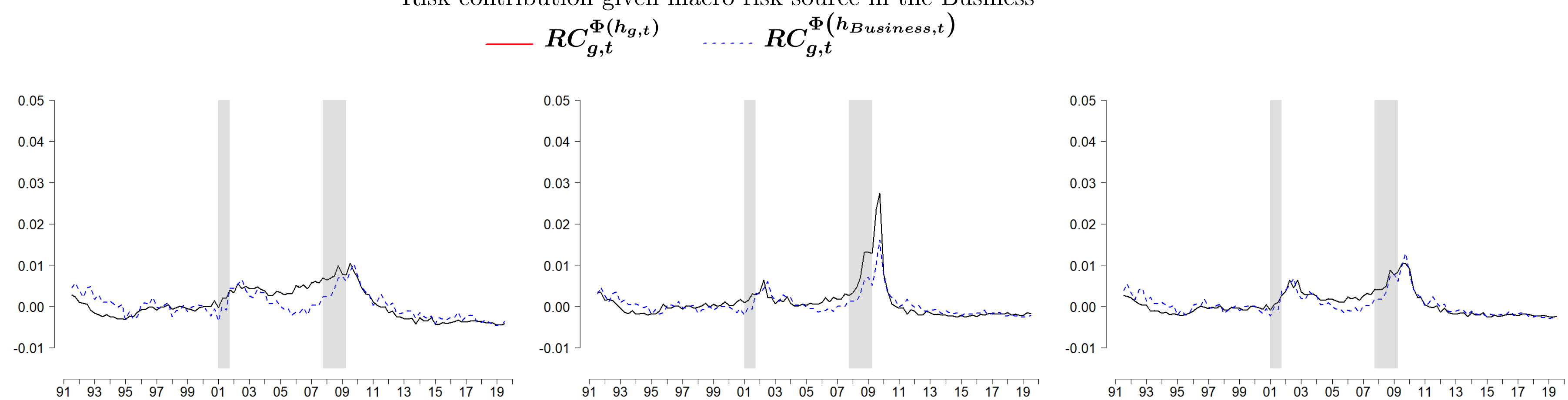
This figure presents the estimated risk sources and their risk contributions for portfolio loss of affected sector g (the Individuals, the Rest, and the Lease) from the causative sector (the Business) by sector. These are represented in pair pictures by sector to compare the effect of macroeconomic and asset correlation by time. The above line presents the risk contributions $RC_{g,t}^{\rho_{g,t}}$ and $RC_{g,t}^{\rho_{Business,t}}$ for sector g given asset correlation within sector $\rho_{g,t}$ or causative sector $\rho_{Business,t}$, respectively in Eq. (16). And, the below line shows the risk contributions $RC_{g,t}^{\Phi(h_{g,t})}$ and $RC_{g,t}^{\Phi(h_{Business,t})}$ for sector g given estimated default thresholds within sector $\Phi(h_{g,t})$ or causative sector $\Phi(h_{Business,t})$, respectively in Eq. (16).

Risk contribution given asset correlation risk source in the Business



(e) Individuals (f) Rest (g) Lease

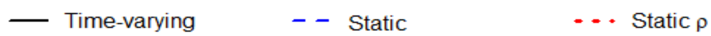
Risk contribution given macro risk source in the Business

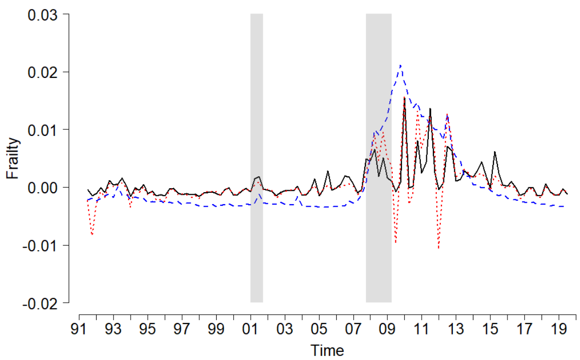


(h) Individuals (i) Rest (j) Lease

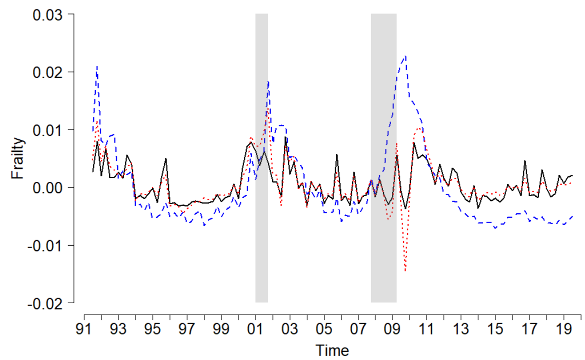
Figure 6: Frailty source contribution by various parameter assumptions model

These figure compares the risk contributions $RC_{g,t}^{frailty}$ of frailty source for each sector portfolio g under various parameters assumption models. The solid line presents the contribution given residual frailty effect $f_{g,t}^{TV}$ in Eq. (13) under time-varying risk sources assumption model in Eq. (8). In contrast, the dash line shows the contribution given estimated f_g^{St} under static assumption model in Section (2.3.1). And, the dot line displays the risk contribution given residual frailty under the partly time-varying parameter (time-varying expected default model Eq. (9) and static asset correlation assumption model). The Creditcards' axis range is marked differently for experession wide range. (eg. Creditcards:-0.02~0.1, others:-0.02~0.03).

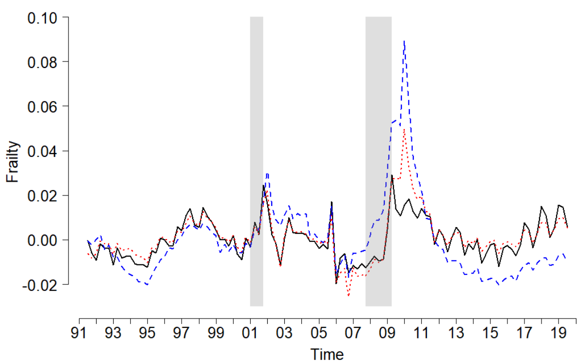




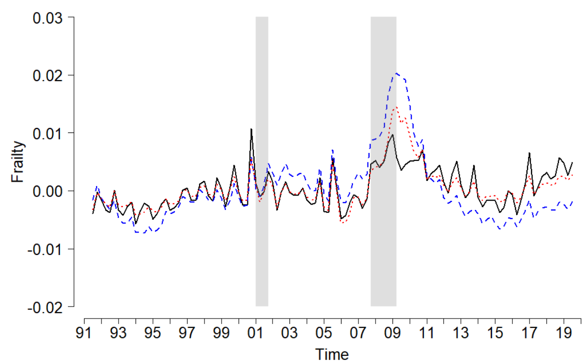
(a) Mortgages



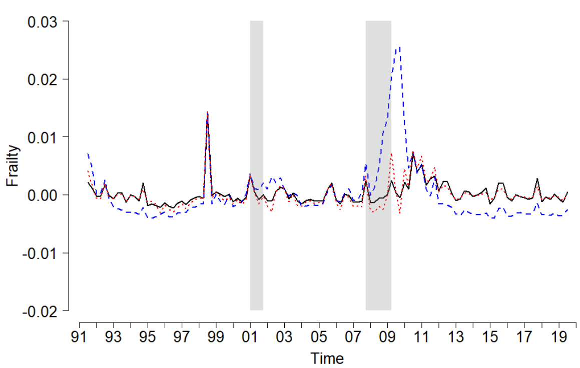
(b) Business



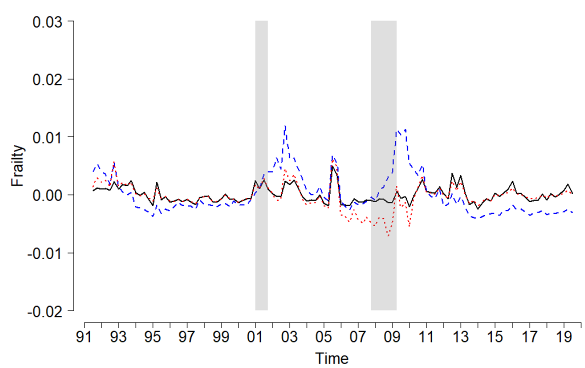
(c) Credit Cards



(d) Individuals



(e) Rest



(f) Lease

Appendix

1. Basel's asset correlation criteria

Table 5: Basel's criteria for asset correlation

This table is the asset correlation criteria employed risk-weighted asset function on Basel committee for banking supervision. (See BCBS (2019))

Exposure class	Loan type	Flexiblilty	Range	Rule
Corp, sovereign	Corporate etc.*	$F(PD)$**	[12%,24%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)$
	FI***	$F(PD)$	[15%,30%]	$1.25 \cdot \left[0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)\right]$
and bank	SMEs****	$F(PD)$	[8%,24%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right) + 0.04 \cdot \left(1 - \frac{(Size-5)}{45}\right)$
	Specialised lending	$F(PD)$	[12%,30%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.30 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)$
	Residential mortgage	Fixed	15%	15%
Retail	Qualifying revolving	Fixed	4%	4%
	Others	$F(PD)$	[3%,16%]	$0.03 \cdot \frac{(1-e^{-35 \cdot PD})}{(1-e^{-35})} + 0.16 \cdot \left(1 - \frac{(1-e^{-35 \cdot PD})}{(1-e^{-35})}\right)$

* Corporates belonging to groups with total considated revenues exceeding EUR 500 million.

**The asset correlation of categories written $F(PD)$ means the function of default probability.

***Financial Institutions.

****Small or Medium-sixed entities

2. Parameter estimation by Maximum likelihood method.

We employ the sequentially follow next four stages to estimate the risk sources. The first stage, in Section 2.2, we have induced the time-varying cumulative distribution function (CDF) Eq. (8) for unconditional portfolio loss by applying the time-varying default threshold in Eq. (9) and the time-varying asset correlation in Eq. (11) to the static risk sources assumption model in Eq (7). Differentiating this cdf with respect to $\ell_{g,t}$ gives the probability densty function (PDF) of time-varying portfolio loss by inverse function theorem as

$$f(\ell_{g,t}) = \sqrt{\frac{1-\rho_{g,t}}{\rho_{g,t}}} \cdot exp\left[\frac{1}{2\rho_{g,t}}\left(h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \tag{17}$$

In the second stage, the risk sources ρ_g and h_g are estimated using the pdf of unconditional portfolio loss cdf in Eq. (7) based on static parameters assumption. The likelihood of the static model given as

$$\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} \sqrt{\frac{1-\rho_g}{\rho_g}} \cdot exp\left[\frac{1}{2\rho_g}\left(h_g - \sqrt{1-\rho_g}\Phi^{-1}(\ell_{g,i})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,i}))^2\right] \tag{18}$$

where $\ell_{g,i}$ is the $i-th$ observable charge-off rate for sector g not considering time steps. These results by portfolio are compared to the time-varying estimates in our empirical analysis in Section 3.2 as static estimates.

In the third stage, we calculate initial values $\rho_{g,t-1}$ of the time-varying model in Eq. (11) by the first 5 quarters data using the static model in Eq. (18) in the second stage. And the initial values β_0 and β_k^z in the time-varying default threshold model in Eq. (9), the sensitivity to economic covariates, are used results of the second stage based on static assumptions.

In the fourth stage, the parameters of the final time-varying model are estimated by the maximum likelihood function in Eq. (19) that contains the time-varying asset correlation $\rho_{g,t}$ in Eq. (11) and the time-varying default threshold $h_{g,t}$ in Eq. (9). The likelihood for time-varying risk sources model is able to write as,

$$\max_{\boldsymbol{\theta}} \prod_{t=1}^{N} \sqrt{\frac{1-\rho_{g,t}}{\rho_{g,t}}} \cdot exp\left[\frac{1}{2\rho_{g,t}}\left(h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \tag{19}$$

where $\boldsymbol{\theta} = (\boldsymbol{\alpha_g}, \boldsymbol{\beta_g}^z)'$ is the parameters set of time-varying models as asset correlation $\rho_{g,t}$ model parameters $\boldsymbol{\alpha_g} = (\alpha_{g,0}, \alpha_{g,1}, \alpha_{g,2})'$ and default threshold $h_{g,t}$ model parameters $\boldsymbol{\beta_g}^z = (\beta_0, \beta_1^z, \beta_2^z, \cdots, \beta_k^z)'$.

The initial values of time-lagged loss distribution $u_{g,t-s}$ in Eq. (11) are used from $F(\ell_{g,t-s}|\rho_{g,t-s}, h_{g,t-s})$

estimated the parameters of the third step based on static assumption models. And the final models are selected by AIC and SBC include optimal persistence time s as short-term shock in time-varying asset correlation model in Section 2.2.2. Through the above four stages, the time-varying $\rho_{g,t}$ and time-varying $h_{g,t}$ can be calculated for each point in time, which means to be able to estimate time-varying loss distribution in Eq. (8). The final estimates of time-varying risk sources display in the empirical results from 1991:Q2 to 2019:Q3, excluding the 5 quarter data used to calculate the initial values in the third stage.