Overconfidence in money management: balancing the benefits and costs

ABSTRACT

Individuals are often overconfident, especially those in positions to influence outcomes. The impact of hiring an overconfident portfolio manager is studied here within the standard principal-agent framework. Overconfidence induces a higher level of effort until the effects of restrictions on portfolio formation emerge. Further, by increasing the incentive fee and sharing more risk, the investor can curb excessive managerial risk taking. When compensation is endogenously determined, we find that investors can benefit from managerial overconfidence. However, excessive overconfidence is detrimental to the investor. In addition, we also find empirical support for some of our model predictions.

JEL classification: G23, M54.
Keywords: Mutual fund, overconfidence, management contracts

Introduction

There is overwhelming evidence that investors are biased and irrational in making investment decisions (see Shleifer (2003), Barberis and Thaler (2003), and Subrahmanyam (2008)). One of the most widely studied behavioral biases is investor overconfidence (see Barber and Odean (2001), Gervais and Odean (2001), and Ben-David et al. (2018)). Although the phenomenon is well documented, most empirical studies and theoretical models limit their focus to overconfidence in individual investors who are not necessarily marginal investors.[1] In this paper, we analyze the decision to hire an overconfident portfolio manager where compensation mechanism and investment constraints can address some of the agency concerns.

It is quite common for financial institutions to delegate the decisions of portfolio formation to professional managers.[2] Delegation is optimal because managers possess superior skills that allow them to collect and process information regarding the movement of security prices. Bhattacharya and Pfleiderer (1985), in their seminal paper, consider the problem of delegation in portfolio management. They propose a compensation contract that screens agents based on their privately known ability. Such a contract also elicits truthful revelation of their private signals from the manager. In this paper, we ask whether screening for overconfident managers is in the best interest of the investor or whether hiring such a manager improves investors' welfare.

We study the above question within the standard principal-agent framework. A risk-averse principal who is aware of the manager's biases sets the contract parameters, and she offers it to the agent. If the contract is feasible to the risk-averse manager, he accepts it and exerts effort. The manager then observes a signal and updates his beliefs about the future return distri-

[1] Palomino and Sadrieh (2011) and Adebambo and Yan (2016) are a few exceptions that deal with overconfidence in investment management.

[2] In what follows, the manager who makes the portfolio decision is also called the agent. The principal who hires the manager is often referred to as the investor.

bution. An overconfident manager updates his beliefs in a biased way and wrongfully estimates the precision of his signal and hence the precision of the ex-post distribution of returns. Based on his beliefs, the manager then makes a decision about the riskiness of the portfolio. We derive the comparative statics of hiring an overconfident manager within this framework. Although contract parameters are endogenously determined, we do not solve for the optimal contract conditional on hiring an overconfident manager. Instead, we evaluate the decision to hire an overconfident manager within the standard compensation structure used in the industry.[3]

We study the problem in two distinct scenarios. First, we solve the problem where there are no constraints on the portfolio holdings of the manager. In this case ("*unconstrained scenario*"), an overconfident manager overestimates the perceived marginal benefit of effort, for any given level of compensation, and will always exert a higher level of effort. Although a perverse result, this scenario captures the essence of hiring an overconfident manager. The principal gains from higher managerial effort and, in addition, can use the incentive parameter in the contract to mitigate excessive risk taking. The optimal effort, however, is not a function of the incentives parameter in the contract (see Stoughton (1993)). Overall, the investor's expected utility is higher from hiring an overconfident manager.

Second, we impose the commonly observed constraints on the manager's portfolio choices ("*constrained scenario*"). A nontrivial 91% of funds face constraints relating to buying on margin, and about 69% of funds do not allow short-selling (see Almazan et al. (2004, Table 1)).[4] Unlike before, when constrained, the optimal effort level is an increasing function of the incentive parameter (see Gómez and Sharma (2006)). Importantly, the equilibrium effort is no longer strictly an increasing function of the manager's overconfidence. First, the overconfident manager overestimates the extent to which

[3] Effort levels are unobservable and hence cannot be contracted.

[4] Dybvig, Farnsworth and Carpenter (2010) also stress the importance of trade restrictions when studying the optimal contract design problem.

his actions influence the final outcome and will put in more effort. Second, for any given signal, an overconfident manager always demands a higher absolute quantity of risky assets. Therefore, when constraints on portfolio formation are imposed, the set of signals for which the manager can demand his utility-maximizing quantity shrinks. He is at the boundary of the allowed quantity more often than a rational manager. Because his additional effort is not rewarded, the manager is bound to reduce his effort level. Therefore, the equilibrium effort increases in overconfidence until it reaches a threshold, after which it decreases. Further, to accommodate managers' bias regarding the marginal benefit of their effort, in our model we allow for the managers' reservation utility to increase in their overconfidence. Overall, it is still beneficial for an investor to hire an overconfident manager up to a certain threshold, beyond which the investor's expected utility diminishes. For robustness, we also solve for the case where the risk preferences of the principal and agent differ and for the case where convex compensation contracts are used.

Conceptually, this paper is similar to Gervais et al. (2011), where overconfident CEOs make investment decisions for the shareholders. The results of Gervais et al. (2011) are similar to the non-monotonic utility gains attributed to hiring an overconfident manager here. The difference is that the focus of the current paper is on delegated portfolio management issues, dealing with the consequences of moral hazard and constraints on portfolio formation. An important advantage of the model presented here is that managerial effort is endogenously chosen, whereas in Gervais et al. (2011), the manager's skill is exogenously specified. Overall, the channels through which the implications of overconfidence are presented here are vastly different from Gervais et al. (2011).

Palomino and Sadrieh (2011) also solve a model of moral hazard where the portfolio manager is overconfident. The focus of their paper, however, is to solve for the optimal contract in which the manager truthfully reveals his signal to the principal. However, their model does not consider restrictions on portfolio formation, something that we think is critical. Moreover, the truth telling contract that Palomino and Sadrieh (2011) propose are not commonly found in the mutual fund industry.

We also empirically test some of the implications of our model. Choi and Lou (2010) provide evidence of self-attribution bias by showing that managers adjust their portfolio holdings and deviate from their benchmarks differently upon receiving news confirming their private signal than upon receiving news that discredits it. Within this framework, we follow Almazan et al. (2004) and create a composite index that quantifies the severity of constraints that managers face. We then show that these constraints significantly mitigate the effect of the above discussed attribution bias. This evidence is in line with our model that shows that portfolio constraints are more binding on an overconfident manager and hence moderate his actions. In a separate specification, we test for the effect of managerial ownership in the fund, our proxy for variable incentive parameter, on the above mentioned bias.[5] As overconfidence increases, the principal must increase the variable component of the contract to increase the variance in the agent's payoff and hence moderate his portfolio decisions. Consistent with this explanation, we empirically find evidence of diminished self-attribution bias in funds having a higher percentage of managerial ownership.

Our main contribution is to help understand the welfare implications of hiring an overconfident manager and the resulting contract design and risk taking in a delegated portfolio management setting. Empirical evidence of self-attribution bias clearly shows the presence of overconfident portfolio managers. The obvious question, then, is why such managers are not screened for. It could mean either that screening mechanisms do not work or that there might be benefits to hiring a moderately overconfident manager. Using a standard model, as in Stoughton (1993), we highlight the gains to hiring

[5]Portfolio manager compensation contracts are private contracts, and hence the contract details are not observable.

an overconfident portfolio manager and explain why such managers continue to exist in equilibrium.

1 Model

The model captures the contracting problem between an investor and a portfolio manager. We abstract away from all other agency problems by assuming that the investment adviser, the board of directors of the fund, and the individual investors are one unit.

1.1 Problem description and preferences

The investor (principal) and the manager (agent) are both risk averse. They are assumed to have a negative exponential utility function where a and b are the absolute risk aversion coefficient of the manager and investor, respectively. Both a and b are non-negative real numbers. The contracting problem begins with the investor seeking to hire a manager who is to employ his skills and extract private signals about future market prices. The investor strategically chooses the contract parameters. In this article, we do not solve for the shape of the optimal contract. Instead, we take the contract form, commonly found in the mutual fund industry, as given and study the choice of contract parameters and evaluate the implications of the hiring decision.[6] The fees have two components: a fixed flat fee, F, and a performance adjustment fee, α. The investor has \$1 to begin with and requires the manager to invest this sum.

The manager has two assets to choose from. He has the option of investing in a risky asset that yields the net return of $\tilde{x}$ or investing in a risk-free asset. The performance adjustment fee is paid when the returns are in excess of

[6] We begin with the linear contract space because most investment advisory contracts are linear. For robustness, we also address the case of convex contracts in the later part of the paper.

a benchmark. The performance fee is assumed to be benchmarked against the risk-free bond, the net return of which is normalized to be zero. After the contract parameters are offered to the manager, the manager decides to accept or reject the contract based on whether his unconditional expected utility meets the reservation utility. The game ends if the manager refuses to accept the contract. Competition in the managerial labor market is not explicitly modeled here. However, the reader could think of the reservation utility as representing the utility from the equilibrium compensation. If the contract is accepted, the manager strategically chooses a level of effort, e, to be exerted. The effort expended allows the manager to observe a random signal , $\tilde{y}$, which is correlated with the future states of the world. After observing the signal, the manager picks the level of risky assets, $\theta(y)$, in his portfolio. All the above decisions are made at the beginning of the period. One period later, the payoffs are realized and contracts are settled. However, no renegotiations are allowed.

Since both the investor and the manager are risk averse, they maximize the expected utility of their respective terminal wealth. The manager will receive a fixed compensation F and a share, α, in the difference between the fund's value, $(1+\theta\tilde{x})$, and the \$1 invested in the risk-free rate. The terminal wealth of the manager is given by

$$\tilde{W}_M(y) = F + \alpha\theta\tilde{x}. \tag{1}$$

Moral hazard in the model is motivated by the fact that unobservable effort is costly and a source of disutility to the manager. The cost function, $V(a,e)$, is a convex increasing function in effort that is twice differentiable. The following cost function is assumed:

$$V(a,e) = ae^2. \tag{2}$$

The terminal wealth of the principal is the value of the portfolio at the end, net of the compensation to the manager. It should be equal to $(1+\theta\tilde{x}) - F - \alpha\theta\tilde{x}$. Ignoring the initial capital, as it does not affect the maximization problem, the following is the terminal wealth of the investor:

$$\tilde{W}_I(y) = (1-\alpha)\,\theta\tilde{x} - F. \tag{3}$$

1.2 Rational and overconfident manager

The prior distribution of the net returns on the risky asset, $\tilde{x}$, is common knowledge and follows a standard normal distribution. The signal, $\tilde{y}$, is assumed to be a noisy indication of the future returns and given as

$$\tilde{y} = \tilde{x} + \tilde{\xi} \tag{4}$$

where $\tilde{\xi}$ is the noise term. It is further assumed that higher levels of effort help in reducing the noise in the signal. In other words, the variance of the noise term decreases with the level of effort; i.e., $\tilde{\xi} \sim N\left(0, \frac{1}{e}\right)$. Stoughton (1993) also shares a similar modeling assumption.[7] Based on these assumptions, for any chosen level of effort, the distribution of the signal is $\tilde{y} \sim N\left(0, \frac{1+e}{e}\right)$. Note, the precision of the signal increases with manager's effort. In this model, the manager is assumed to be Bayesian. So, after observing the signal, the manager updates his beliefs about the distribution of the risky asset's return[8] to

$$\tilde{x}|y \sim N\left(\frac{e}{1+e}y, \frac{1}{1+e}\right). \tag{5}$$

The above beliefs are that of a rational manager. An overconfident manager believes that, for any level of effort he chooses, the following is the distribution of the noise in his signal:

[7]Although productivity of effort depends on skill, to make the larger point, we assume that managerial skill is cross-sectionally the same.

[8]This is the conditional normal distribution of the returns given effort and the signal.

$$\tilde{\xi}_\psi \sim N(0, \frac{1}{\psi e}) \tag{6}$$

where $\psi \geq 1$ is the level of overconfidence. A higher ψ implies that the agent is more overconfident. In the case when $\psi = 1$, we return to the rational world. An overconfident manager assumes that his effort reduces the variance in the noise term much more than a rational manager does.[9]

One possible criticism of the above setup is that overconfidence is exogenously specified. To mitigate this concern, the reader should think of this model as one period in a multi-period setup where nobody, including the manager, knows the true ability of the manager. They update their beliefs about his ability after every round of trading. The overconfident manager updates his belief in a biased way such that he takes undue credit in instances of success but fails to take proportional responsibility for failure. The investor, however, rationally updates her beliefs about the manager. Gervais and Odean (2001) show that this mechanism, referred to as self-attribution bias, endogenously leads to overconfidence. Therefore, the model presented here is just the nested version of the above described framework. This abstraction is helpful because we use a simple model to present important effects of managerial overconfidence. What really matters for the analysis is that there is heterogeneity in beliefs; the source of it is less relevant. Given the above beliefs, the overconfident manager will have the following as the conditional distribution for asset return:

$$\tilde{x}|y \sim N\left(\frac{e\psi}{1+e\psi}y, \frac{1}{1+e\psi}\right). \tag{7}$$

[9]Manifestation of overconfidence happens in three distinct ways: *overestimation*, *overplacement*, and *overprecision* (Moore and Healy (2008)). Our modeling assumption is more akin to *overestimation,* where the manager overestimates his ability or the value of his effort. Below, you will see that our modeling choice affects not only the mean but also the standard deviation of the conditional return distribution (*overprecision*).

1.3 Unconstrained problem

To solve her problem, the investor must first solve the manager's problem. Here, in the first case, the manager strategically chooses a level of effort and the quantity of the risky asset in an unconstrained way. For any given level of effort and signal, the manager will maximize his conditional utility by choosing a utility maximizing quantity. Solving the manager's utility maximization problem, we get the following expression for the optimal quantity demanded:

$$\theta = \frac{e\psi}{a\alpha} y. \tag{8}$$

All proofs are provided in the appendix. First, the proportion of wealth invested in the risky asset is an increasing function of the manager's overconfidence and effort and, understandably, decreases in the level of his risk aversion. Second, as expected, a higher positive signal implies that a larger proportion of the wealth is invested in the risky asset.

The next step in this method of backward induction is to solve for the agent's equilibrium effort. The manager has to weigh the marginal benefit of effort, which is a higher signal precision, against the marginal cost of effort. The following equation represents the unconditional expected utility function of the manager:

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}.g(e) \tag{9}$$

where $g(e) = \left(\frac{1}{1+e\psi}\right)^{\frac{1}{2}}$. It is evident from the above equation that the manager's expected utility increases in the function $g(e)$. Note that in equation (9), the unconditional expected utility of the manager is not a function of the incentive parameter α. Stoughton (1993) was the first to show that linear contracts cannot be used to induce higher effort from the manager. Overall, the optimal effort that maximizes the manager's expected utility should solve the following first-order condition:

$$V'(a, e_{fb}) = \frac{\psi}{2}(\frac{1}{1 + \psi e_{fb}}). \tag{10}$$

Because the expected utility function does not depend on α, the optimal effort also will not be a function of the incentive parameter. The question that is of interest to this paper is the response in effort choice to changes in the level of overconfidence. Because the overconfident manager thinks that the precision of his signal is high, he is bound to overestimate the marginal benefit of his effort. Therefore, the point of indifference between the marginal utility of effort and the marginal cost of effort will be at a higher effort level than it is for a rational manager.[10]

Proposition 1. *Given any contract (α, F), the optimal effort, e_{fb}, of the manager increases with overconfidence, ψ.*

The fact that the effort increases with overconfidence also has to do with the modeling assumptions of complementarity between effort and overconfidence. Although in some instances they could be thought of as substitutes, Bénabou and Tirole (2002) argues that substitutability typically occurs when the reward for performance is of a "pass-fail" (binary) nature, which is not the case here. The manager faces a continuum of outcomes on the return distribution.

Since the investor in the model is rational, the distributions used in computing her expected utility are those of a rational person. In deriving the investor's expected utility function, we define the following two functions:

$$m(\alpha) = \frac{(1 - \alpha)}{\alpha}\psi,$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)). \tag{11}$$

[10]Solving equation (10), the optimal effort (e_{fb}) is equal to $\frac{-a+\sqrt{a\psi^2+a^2}}{2\psi a}$. This effort function is positive and increases with ψ.

The following is the investor's unconditional expected utility:

$$E_i(U) = -\exp\{aF\}\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}. \tag{12}$$

Along with satisfying the incentive compatibility constraint in equation (10), the investor also has to ensure that the minimum reservation utility, U_o, is met to secure the manager's participation. The following is the participation constraint:

$$-\exp\{-aF + V(a, e_{fb})\}\left(\frac{1}{1+e_{fb}\psi}\right)^{1/2} = -U_o. \tag{13}$$

Above, we assume that the investor and the manager have the same level of risk aversion a. This is done to study the effects of overconfidence in isolation and to exclude any confounding effects arising from the differences in the agent's risk preferences. We relax this constraint below and show the effects of differences in risk preferences.

Lemma 1. *In the unconstrained scenario, for a given level of managerial overconfidence ψ, the investor chooses*

$$\alpha_{fb} = \frac{\psi}{1+\psi} and$$

$$F = \left(\frac{1}{a}V(a, e_{fb}) + \frac{1}{2a}\log\left(\frac{1}{1+e_{fb}\psi}\right) - \frac{1}{a}\log(U_o)\right)$$

as the contract parameters.

The potential benefits of hiring an overconfident manager are presented below.

Proposition 2. *The expected utility of a rational investor always increases with the level of managerial overconfidence.*

Intuitively, because, for any given contract, an overconfident manager will always choose a higher equilibrium effort, there should be benefits from hiring an overconfident manager. However, an overconfident manager, because of his bias, will always pick a riskier portfolio for any given signal; i.e., when compared to that of a rational person (see equation (8), $\theta(y)$ increases with ψ). However, by choosing an appropriate α, the principal can address this problem and implicitly choose the level of portfolio risk. To see this, compute the quantity of the risky asset that the principal will demand in the event that she observes the signal herself. Given her expected utility function, the optimal quantity, based on rational beliefs, is the following:

$$\theta_i(y) = \frac{e}{a(1-\alpha)} y. \tag{14}$$

When $\alpha = \frac{\psi}{1+\psi}$, as in Lemma 1, $\theta_i(y) = \theta$, the exact quantity that the manager will pick. Higher equilibrium effort and the ability to weigh in on the extent of portfolio risk, using the variable compensation parameter, ensures that it is always optimal for the investor to hire an overconfident investor in the unconstrained case.

Daniel et al. (1998), through their model, argue that when individuals/traders are overconfident they trade more often and hold riskier positions. Barber and Odean (2001) and Grinblatt and Keloharju (2009) provide empirical support for these claims. We also support these findings.

Proposition 3. *The quantity of the risky asset demanded by an overconfident agent is always higher than the quantity demanded by a rational manager.*

Whether the additional risk in the portfolio generates higher returns is an empirical question. However, it is important to note that the outcome of Proposition 3 is optimal from the principal's perspective.

1.4 Constrained problem

Portfolio managers often do not make decisions in an unconstrained way, as was depicted in section 1.3. Almazan et al. (2004) reports that a vast majority of funds have a variety of constraints on the portfolio holdings. Regardless of the source of the constraint, imposing such constraints can have profound effects on contracting decisions (see Dybvig et al. (2010)).

We follow Gómez and Sharma (2006) and model the constraint by restricting the absolute value of the amount of the risky asset demanded to a positive constant k in the following way:

$$|\theta(y)| \leq k. \tag{15}$$

The value of k is exogenously specified and used here just to illustrate a point. As k tends toward infinity, we would be back to the case of no constraints. The demand function for the quantity of the risky asset is no longer a smooth function as in the unconstrained case. Instead, we now have a piecewise function depending on the value of k:

$$\theta(y) \begin{cases} k & y > \frac{ka\alpha}{\psi e} \\ \frac{\psi e}{a\alpha} y & |y| < \frac{ka\alpha}{\psi e} \\ -k & y < -\frac{ka\alpha}{\psi e}. \end{cases} \tag{16}$$

The manager can get the quantity of his choice as long as that quantity corresponds to the signal in $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$. However, for any signal outside this range, i.e., $y < -\frac{ka\alpha}{\psi e}$ and $y > \frac{ka\alpha}{\psi e}$, the quantity demanded is restricted to $-k$ and k, respectively.

1.4.1 Manager's problem

From the manager's demand function, we can derive his unconditional expected utility function.

Lemma 2. *The unconditional expected utility function of the manager is given by*

$$E_m[\mathrm{U}] = -exp(-aF + V(a,e)).g(e,\psi|\alpha), \tag{17}$$

with

$$g(e,\psi|\alpha) = (\frac{1}{1+\psi e})^{\frac{1}{2}}\Phi(\frac{(ka\alpha)^2}{\psi e}) + exp(\frac{(ka\alpha)^2}{2})(1-\Phi(\frac{(ka\alpha)^2}{\psi e}(1+\psi e)))$$

where Φ *is the distribution function of a* $\chi^2(1)$ *random variable.*

The function $g(e,\psi|\alpha)$ now has two distinct components. The first component corresponds to the set of signals within the bounds where the overconfident manager is not affected by this constraint. The second term relates to signals where the constraint is binding. The manager's expected utility function leads to the following first-order condition for effort:

$$V'(a,e^*)g(e^*,\psi|\alpha) + g'(e^*,\psi|\alpha) = 0 \tag{18}$$

where

$$g'(e,\psi|\alpha) = \frac{-\psi}{2}\left(\frac{1}{1+\psi e}\right)^{3/2}\Phi\left(\frac{(ka\alpha)^2}{\psi e}\right). \tag{19}$$

The key distinction here, from the unconstrained case, is that $g(e,\psi|\alpha)$ now depends on α. This is one of the main contributions of Gómez and Sharma (2006).

Lemma 3. *The manager's equilibrium constrained effort increases with performance adjustment fee* α*.*

In the presence of a constraint on the quantity demanded, two opposing forces influence the choice of effort for an overconfident manager. As shown in the unconstrained case, overconfidence leads to an increase in the amount of effort because the manager perceives the marginal benefit of his effort to be high. However, in the presence of constraints, the signal space for which the manager can choose the utility maximizing quantity decreases with the level of his overconfidence. This is evident from the fact that the measure of the set $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$ decreases as overconfidence, ψ, increases. This implies that the unconditional probability of the manager being forced to pick quantities k or $-k$ is higher when the manager is overconfident than when he is rational. As this is bound to decrease the overconfident manager's expected utility, his intuitive response is to then reduce the effort ex-ante. The tradeoff between these two effects will determine the equilibrium level of effort. This line of thought also provides an economic explanation for Lemma 3. As α increases, the measure of set $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$ also increases, meaning that the set of possible signals for which the manager can pick his optimal quantity increases. This in turn raises his expected utility and hence induces higher effort.

Unfortunately, it is extremely hard to compute an analytical expression for the level of effort from (18). It is also not feasible to do any comparative statics given that we already expect a non-monotonic relationship between effort and overconfidence. Therefore, we present a numerical solution for the choice of effort.

Proposition 4. *Due to higher perceived precision by the manager, the constrained optimal effort increases with overconfidence up to a certain point. However, as overconfidence increases beyond this level, it has a negative impact on effort.*

Figure 1 plots the optimal effort as a function of overconfidence. The plot is generated by assuming values of 1 and 0.2 for k and α, respectively.[11] We follow Haubrich (1994) and set $a = 1.25$ because it explains the empirical pay-performance relationship in CEO compensation. The concave down relationship between effort and overconfidence meets the expectation presented earlier regarding the two opposing effects of overconfidence.

[11] The results are qualitatively similar to those of other choices of parameters.

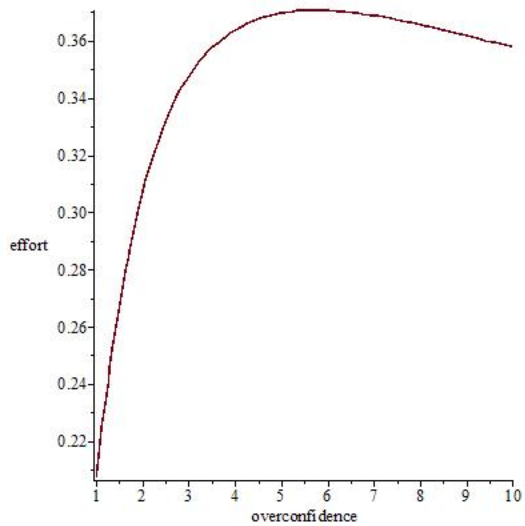

Figure 1: Optimal level of effort chosen by the manager as a function of the level of his overconfidence. Equation (18) presents the first-order condition for effort. It has been solved for effort by assuming values of 1, 1.25, and 0.2 for k, a, and α, respectively, at different levels of overconfidence.

1.4.2 Investor's problem and numerical results

Earlier, in the unconstrained case, it is assumed that all managers have the same reservation utility. However, if an overconfident manager perceives the marginal benefit of his effort to be high, then he, arguably, could also demand higher compensation. To address this, we allow the reservation utility of the manager to be an increasing function of his overconfidence. The following is the new participation constraint:

$$-exp(-aF + V(a,e)).g(e,\psi|\alpha) \geq -exp(-a \cdot r(\psi)), \tag{20}$$

where $r(\psi)$ is the reservation wealth of the agent. To compute the contract parameters, we have to specify the investor's objective function.

Lemma 4. *The investor's constrained unconditional expected utility function is given by*

$$E_i\left(U\right) = -exp\{aF\} \cdot f(\alpha, e), \tag{21}$$

where

$$f(\alpha,e)=\left(\frac{1}{1+eM\left(\alpha\right)}\right)^{\frac{1}{2}}\Phi\left(\frac{\left(ka\alpha\right)^{2}}{e\psi^{2}}\frac{1+eM\left(\alpha\right)}{1+e}\right)$$
$$+$$
$$exp(\frac{\left(ka\left(1-\alpha\right)\right)^{2}}{2})\left(1-\Phi\left(\frac{\left(ka\alpha\right)^{2}}{e\psi^{2}}\frac{\left(1+em\left(\alpha\right)\right)^{2}}{1+e}\right)\right).$$

Φ above is the distribution function of a $\chi^2(1)$ random variable. Given the nature of the above expression, there are no closed-form solutions for the contract parameters α and F. Therefore, we explore numerical solutions. Note that the agent's choice of effort is not a function of the fixed compensation F, a standard result in most principal-agent models. Additionally, from (21) we know that the investor's expected utility decreases with F and from (20) that the manager's expected utility increases with F. This means that the participation constraint has to be binding at the optimum. So the investor's problem can be reduced further to make it a function of only one choice variable α in the following way:

$$E_i\left(U\right)=-exp\{V(a,e)+a\cdot r(\psi)\}\cdot g(e,\psi|\alpha)\cdot f(\alpha,e). \tag{22}$$

Importantly, by changing α, the investor can control managerial effort and moderate risk taking. To execute the numerical procedure, k is set to 1 for all further numerical computations. The algorithm starts by creating a grid for the possible values of the incentive parameter α, i.e., between 0 and 1. In each iteration, one of the possible hundred values of α is selected. Conditional on the chosen α, the next step involves solving the manager's problem and evaluating the optimal effort. Subsequently, for each pair of (α, e) and the given level of overconfidence, the investor's expected utility is computed using (22). Having evaluated the expected utility of the investor over all the possible values of α, the final step is to choose the α that provides the maximal expected utility. This procedure is then repeated for multiple levels of managerial overconfidence. The results of the numerical computations are reported in Table 1. Based on these findings, the following proposition is in order.

Proposition 5. *Assuming a symmetric linear compensation structure,*
a) It is always beneficial for the risk-averse investor to hire a moderately overconfident manager in the constrained case.
b) The level of portfolio risk is higher when an overconfident manager is hired.

Panels A - D of Table 1 present results to support the above proposition. The four different panels report results for each of the different assumptions regarding the reservation wealth. In Panel A, it is assumed that the rational manager, $\psi = 1$, desires 1% of assets under management (AUM) as reservation wealth. It is further assumed that the manager's expectation increases linearly in ψ: e.g., $0.1 \times \psi \times AUM$. Similarly, Panel B assumes that a rational manager expects to earn 3% of the assets as fees. In Panel C, risk aversion parameter is changed. Finally, in Panel D, we assume that the expected reservation wealth increases quadratically. The first row in each of these panels reports the investor's expected utility (IEU) from hiring managers of different overconfidence levels. In Panels B, C, and D, the investor's expected utility increases with overconfidence up to a point and then decreases subsequently. There are two main effects of overconfidence. First, it increases the level of equilibrium effort, causing the mean and the precision of conditional return to go up. Second, the overconfident manager will hold a riskier portfolio than appropriate conditional on his signal. This decreases the investor's expected utility as it increases the variance of the portfolio. Traditionally, the compensation contract is used to trade off between incentives and insurance. The principal's rational response will be to increases the incentive parameter and share more risk with the agent. The manager's participation is ensured because of his bias and the associated increase in expected utility. Increasing

α also reduces the quantity of the risky asset demanded (see equation (16)). Therefore, overall, one expects to see α increase with overconfidence.

The second row in all the panels of Table 1 details the amount of money invested in risky assets.[12] As expected, overconfident managers invest a higher proportion of wealth in the riskier asset. As overconfidence increases, it becomes increasingly expensive for the investor to ensure participation, so the shape of the expected reservation wealth function determines the degree of trade-offs. The final row in Table 1 shows that the equilibrium effort chosen increases with overconfidence. This should not be construed as a violation of Proposition 4, which holds only *ceteris paribus.* As the incentive parameter changes with each level of overconfidence, so does the effort. Note that the first-order condition for effort choice, equation (18), is not a function of the reservation utility, $r(\psi)$. Therefore, the optimal effort is the same in Panels A, B, and D.

2 Role of risk aversion

In our analysis thus far, we assumed that the investor and the manager have the same risk aversion levels. To highlight the role of overconfidence and heterogeneity in beliefs, it was imperative to eliminate the effects, if any, of the differences in risk aversion on effort choice and portfolio formation. Here, we explore the contracting problem by allowing risk aversion levels to differ. It is standard to assume that the manager is more risk averse than the investor, who represents the group of investors.

From Grossman and Hart (1983), we already know that when the agent has a CARA utility function, the loss to the principal on account of the moral hazard increases with the agent's degree of absolute risk aversion. Therefore, a priori, the expectation is that the investor's expected utility should decrease

[12] The proportion of wealth invested in the risky asset is contingent on the signal. "$ in risky" reported here is in expectational terms.

Table 1: Investor's Expected Utility in Constrained Case

Results from the numerical computations for the constrained case are reported here. The investor's expected utility, IEU, is computed using equation (22). Details of the exact algorithmic procedure are presented in the main text of the paper. \$ in risky is the expected % of initial capital that is invested in the risky asset by the manager. The performance adjustment fee, α, is the value of the optimal contract parameter chosen by the investor. Effort, e, is endogenously chosen by the manager given the contract parameters. The degree of portfolio constraints is uniformly set, $k = 1$. The values are reported for different levels of overconfidence, ψ. Panel A reports the values assuming that the reservation wealth of the manager is 1% of AUM and linearly increases with overconfidence. The values are reported under the assumption that the absolute risk aversion parameter for both the agents is 1.25. In Panel B, the reservation wealth of the manager is assumed to be 3% of AUM. In Panel C, the values are reported assuming a risk aversion parameter of 2. In Panel D, it is assumed that the reservation wealth increases quadratically with overconfidence.

	Value of overconfidence (ψ)								
	1	1.5	2	2.5	3	3.5	4	4.5	5
	Panel A: risk aversion parameter $a = 1.25$ - Linear 0.01								
IEU	-0.908	-0.875	-0.850	-0.830	-0.814	-0.800	-0.788	-0.779	-0.771
\$ in risky	0.485	0.559	0.617	0.661	0.693	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254
	Panel B: risk aversion parameter $a = 1.25$ - Linear 0.03								
IEU	-0.931	-0.909	-0.894	-0.884	-0.877	-0.873	-0.872	-0.872	-0.874
\$ in risky	0.485	0.559	0.617	0.662	0.694	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254
	Panel C: risk aversion parameter $a = 2$ - Linear 0.03								
IEU	-0.979	-0.965	-0.954	-0.947	-0.943	-0.942	-0.945	-0.949	-0.955
\$ in risky	0.281	0.347	0.409	0.464	0.515	0.554	0.587	0.617	0.642
α	0.5	0.6	0.66	0.7	0.72	0.74	0.76	0.77	0.78
Effort	0.112	0.151	0.179	0.199	0.212	0.222	0.229	0.234	0.238
	Panel D: risk aversion parameter $a = 1.25$ - Quadratic 0.01								
IEU	-0.908	-0.883	-0.872	-0.870	-0.877	-0.893	-0.916	-0.949	-0.990
\$ in risky	0.485	0.559	0.617	0.662	0.694	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254

as the manager's risk aversion increases. We follow the numerical procedure detailed in Section 1.4.2 and analyze the problem when there are differences in the risk aversion levels.

Having differences in the risk aversion levels will not affect the manager's problem. However, the investor has to solve the following equation instead of equation (22):

$$E_i\left(U\right) = -exp\{V(a,e)\frac{b}{a} + b \cdot r(\psi)\} \cdot g(e,\psi|\alpha)^{\frac{b}{a}} \cdot f(\alpha,e), \tag{23}$$

where

$$m(\alpha) = \frac{b}{a}\frac{(1-\alpha)}{\alpha}\psi,$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)),$$

and

$$\begin{aligned} f(\alpha,e) = \left(\frac{1}{1+eM\left(\alpha\right)}\right)^{\frac{1}{2}} \Phi\left(\frac{\left(ka\alpha\right)^2}{e\psi^2}\frac{1+eM\left(\alpha\right)}{1+e}\right) \\ + \\ exp(\frac{\left(ka\alpha m\left(\alpha\right)\right)^2}{2\psi^2})\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{e\psi^2}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right). \end{aligned}$$

Table 2 presents the results in a manner similar to that presented in Table 1. The results are presented in a way that facilitates easy comparison. The only way that Panel A of Table 2 differs from Panel B of Table 1 is that it assumes that the manager's risk aversion coefficient, a, is 2.5 instead of 1.25. Comparing these two tables, one can observe that the investor's expected utility (row 1) is lower for all levels of overconfidence when the manager's risk aversion is higher. A similar conclusion can be drawn by

Table 2: Investor's Expected Utility - Differences in Risk Aversion.

Results from the numerical computations for the second-best case are reported here. The investor's expected utility, IEU, is computed using equation (22). Details of the exact algorithmic procedure are presented in the main text of the paper. $ in risky is the expected % of initial capital that is invested in the risky asset by the manager. The performance adjustment fee, α, is the value of the optimal contract parameter chosen by the investor. Effort, e, is endogenously chosen by the manager given the contract parameters. The degree of portfolio constraints is uniformly set, $k = 1$. The values are reported for different levels of overconfidence , ψ. Panel A reports the values assuming that the reservation wealth of the manager is 3% of assets under management, and it linearly increases with overconfidence. The values are reported under the assumption that the absolute risk aversion parameter for the investor is 1.25 and that for the manager is 2.5. In Panel B, reservation wealth of the manager is assumed to be 1% of assets under management and that it increases quadratically with overconfidence. As in Panel A, the absolute risk aversion parameter for the investor is assumed to be 1.25 and that for the manager to be 2.5.

	Value of overconfidence (ψ)								
	1	1.5	2	2.5	3	3.5	4	4.5	5
Panel A: risk aversion parameter $a = 2.5$, $b = 1.25$ - Linear 0.03									
IEU	-0.983	-0.975	-0.971	-0.969	-0.970	-0.973	-0.977	-0.983	-0.990
Effort	0.091	0.125	0.150	0.169	0.183	0.194	0.201	0.208	0.212
α	0.34	0.43	0.5	0.54	0.58	0.6	0.62	0.64	0.65
$ in risky	0.295	0.346	0.390	0.439	0.476	0.516	0.548	0.574	0.601
Panel B: risk aversion parameter $a = 2.5$, $b = 1.25$ - Quadratic 0.01									
IEU	-0.958	-0.948	-0.947	-0.954	-0.970	-0.994	-1.027	-1.069	-1.121
Effort	0.091	0.125	0.150	0.169	0.183	0.194	0.201	0.208	0.212
α	0.34	0.43	0.5	0.54	0.58	0.6	0.62	0.64	0.65
$ in risky	0.295	0.346	0.390	0.439	0.476	0.516	0.548	0.574	0.601

comparing Panel D of Table 1 and Panel B of Table 2, where the expected reservation wealth increases quadratically. Other values for the manager's risk aversion are also tried, and the results are qualitatively similar. These results confirm our earlier intuition about the effects of differing risk aversion levels. Importantly, the investor's expected utility still increases from hiring a manager who is moderately overconfident.

3 Convex contracts, including hedge funds

Another possible limitation of our model is that it assumes linear contracts. In the mutual fund industry, it is common practice for investment advisers to provide convex or asymmetric contracts to portfolio managers (see Ma et al. (2019) and Lee et al. (2019)). This structure implies that incentive fees are paid when the fund earns a positive return but no money is deducted in the event of negative returns. Hedge funds also use a similar type of compensation contract.[13] Given this compensation structure, is it still profitable to hire an overconfident manager?

In the spirit of our main analysis, we present a simple two-state model for the case where the manager has a convex contract. Using this setup, we show that the earlier arguments, made using linear contracts, continue to hold. Consider a two-state economy where the risky asset could either return x_1 or $-x_1$, where $x_1 > 0$. The prior probability is that the two states are equally likely. Similar to the earlier setup, the manager now exerts effort and observes a signal regarding the future returns of the risky asset. If the manager observes the signal, s_1, then the probability of the future return being x_1 is given by $p(\psi, e)$. Assume that the posterior probability is given by

$$p(\psi, e) = \frac{1}{2} + \frac{1}{2}\frac{\psi}{1+\psi}\frac{e}{1+e}. \tag{24}$$

The posterior probability, $p(\psi, e)$, increases with overconfidence, ψ, and in effort, e, and is higher than the prior probability of 0.5. This would also imply that the probability of $-x_1$ given s_1 is less than 0.5. The wealth of the manager in the two states would be given by

[13] Hedge funds use a fee structure that is commonly referred as two and twenty fees. More specifically, the manager earns 2% of total asset value as a management fee and an additional 20% of any profits earned.

$$\tilde{W}_M = \begin{cases} F + \beta x_1 \theta & with\, probability\, p \\ F & with\, probability\, (1-p) \end{cases}$$

where F is the fixed fee, β is the incentive fee, and θ is the quantity of the risky asset demanded. The difference in the payoffs of the two states showcases the convexity in the compensation.

A portfolio manager with a CARA utility function (like the negative exponential) and normally distributed returns is similar to a mean-variance maximizer. Therefore, we assume that the expected utility of the manager with terminal wealth, $\tilde{W}$, is given by

$$E_m\,(U) = E(\tilde{W}) - \frac{1}{2}\,a\,Var(\tilde{W}) - V(a,e). \tag{25}$$

The first step in solving the manager's problem is to compute the proportion of wealth invested in the risky asset. The optimal amount of the risky portfolio is given by

$$\theta = \frac{1}{a\,(1-p)x\,\beta}. \tag{26}$$

The proof of the above follows much like the proof provided in the appendix, section A.1. The quantity of the risky asset demanded is contingent on the signal that is observed. $\frac{1}{a\,(1-p)x_1\,\beta}$ and $-\frac{1}{a\,(1-p)x_1\,\beta}$ are the amounts of the risky asset demanded when the signals are s_1 and $-s_1$, respectively. Further, because $p(\psi, e)$ increases with both overconfidence and effort, it is easy to see that the amount invested in risky assets also increases with ψ, and e.

Having solved the investment problem, the expected utility of the manager (given effort) will be

$$E_m(U|e) = F + \frac{p}{2a\,(1-p)}. \tag{27}$$

Equation (27) can be further used to compute the optimal effort expended by the manager. The first-order condition for effort is the following:

$$\frac{1}{a}\frac{\psi(1+\psi)}{(1+e+\psi)^2} - 2ae \; = 0. \tag{28}$$

The crucial relationship is the one between managerial overconfidence and the level of effort chosen. We define the left hand side of the above first-order condition, equation (28), as H. The partial derivatives of H with respect to ψ and e are given by

$$\frac{\partial H}{\partial \psi} = \frac{1+\psi+e+2\psi e}{(1+\psi+e)^3} > 0, \; and$$

$$\frac{\partial H}{\partial e} = \frac{-2\psi(1+\psi)}{a(1+\psi+e)^3} - 2a < 0.$$

Using the implicit function theorem, we can conclude that managerial overconfidence increases the endogenously chosen effort level ($\frac{\partial e}{\partial \psi} > 0$) in the current case of an unconstrained manager having a convex payoff. This result combined with the implications of equation (26) ensures that the results presented in the constrained case (Section 1.4) also hold for a manager with convex compensation.

4 Empirical analysis

In this section, we use the model insights and empirically test some of our predictions. It is very hard to get any data on the direct assessment of managerial overconfidence.[14] Gervais and Odean (2001), however, show that

[14]Grinblatt and Keloharju (2009), to the best of our knowledge, is the first and only study to employ direct psychological assessment of overconfidence among traders. The

biased attribution of outcomes to one's ability leads to overconfidence. Biased attribution occurs when a Bayesian manager updates his priors about his private signal more aggressively after receiving evidence confirming his earlier signal than he does after receiving refuting subsequent evidence. Using a stylized model, Choi and Lou (2010) show that biased attribution on the part of portfolio managers affects subsequent portfolio choices. Within the framework of their model, Choi and Lou (2010) establish that the number of positive/confirming signals the managers receive, measured by the sum of positive benchmark-adjusted return (SPR), is positively related to the manager's portfolio risk choices, measured as the sum of absolute deviations from one's benchmark index ($Active\,Share$).

We borrow the empirical design in Choi and Lou (2010) to test their predictions and confirm the existence of such a bias. Further, in section 1.4, we have clearly shown the importance of portfolio constraints and how they are more binding on overconfident managers. The presence of the portfolio constraints ex-ante lowers managerial effort, the precision of the signal, and hence the extent of portfolio deviation. Of course, this is after controlling for the direct effect of constraints on such deviations.

Hypothesis 1: The effect of confirming positive signals on future portfolio deviations diminishes in the presence of more portfolio constraints.

Endogoneity of compensation and the ability to constrain managers are the main reasons that make it feasible to hire moderately overconfident managers. Lemma 1 and the results of Table 1 clearly show that as managerial overconfidence increases, the variable compensation needs to increase with it. More variable compensation increases the risk sharing and also moderates the overconfident manager's portfolio decisions. Fund managers often have

data on traders' psychological evaluation are available because standard psychological assessments are performed on all Finnish males at the time of their induction into mandatory military service.

ownership in the fund that they manage, and as this ownership increases, it increases the variability of their payoff.

Hypothesis 2: The effect of confirming positive signals on future portfolio deviations diminishes in the presence of higher managerial ownership in the fund.

4.1 Mutual fund and benchmark index data

Data for the empirical analysis are drawn from two main sources. First, we extract mutual fund holdings data from the Thomson Reuters Mutual Funds Holdings database from 2000 to 2014. Most funds in this database file a quarterly holdings report. Moreover, the filing date (fdate) is often different from the report date for which the holdings are valid (rdate), and the reported number of shares in Thomson is split-adjusted as of the filing date. Following Choi and Lou (2010), we reverse the adjustment process done by Thompson because we need to compute the number of shares held on the report date.

Our second source of data is the Center for Research in Security Prices (CRSP) Mutual Fund database, which includes fund characteristics, net asset values (NAVs), and returns for each share class. Although all these information is provided at the share class level, the underlying portfolio for the different share classes within a fund is the same. Therefore, to aggregate data at the fund level, we use the MFLINKS data provided by Wharton Research Data Services (WRDS). Fund's expense ratio and turnover ratio are the weighted averages of the ratios of its different share classes. The weights are based on the total net assets (TNA) of each share class at the beginning of the period. Finally, we merge the Thompson Reuters Mutual Funds Holdings database with the CRSP Mutual Fund database using MFLINKS data. We remove index funds from the sample by removing funds that have *index*, *indx*, and *idx* in their names.

In our analysis, following Cremers and Petajisto (2009), we consider 19 widely used indices from three major U.S. index families. From the S&P indices, we pick the S&P 500, S&P500/Barra Growth, S&P500/Barra Value, S&P MidCap 400, and S&P SmallCap 600. From the Russell indices, we pick Russell 2000, Russell 2000 Growth, Russell 2000 Value, Russell 1000, Russell 1000 Growth, Russell 1000 Value, Russell 3000, Russell 3000 Growth, Russell 3000 Value, Russell Midcap, Russell Midcap Growth, and Russell Midcap Value. Finally, from Wilshire indices, we pick Wilshire 4500 and Wilshire 5000. Index constituents data are obtained directly from the companies that manage those indices.

4.2 Active share and delta active share

We are interested in documenting the effect of observing public signals on changes in the deviation of the portfolio from its benchmark. According to Cremers and Petajisto (2009), Active Share is defined as one half of the sum of absolute deviations in the portfolio weight of the mutual fund from its benchmark index:

$$ActiveShare = \frac{1}{2} * \sum_{N=1}^{N} |w_n^{fund} - w_n^{index}| \tag{29}$$

where w_n^{fund} and w_n^{index} are the portfolio weight of stock n in the fund and that of each constituent in the fund's benchmark index, respectively. The sum is taken over the universe of all assets. Following Cremers and Petajisto (2009), we compute the Active Share of a fund with respect to nineteen indices and assign the index with the lowest Active Share as the fund's benchmark. This index has the largest overlap with the fund holdings.

However, one limitation of the Active Share is that there can be an artificial variation in the Active Share when a stock price changes are not accompanied by the actual trading of fund holdings. Choi and Lou (2010) effectively circumvent such mechanical variation by using the Delta Active Share (ΔAS). The Delta Active Share is defined as the Active Share of a fund at the end of quarter t minus the Active Share of a hypothetical portfolio if a manager does not trade during the quarter t. The Delta Active Share will be zero if the portfolio weights of funds holdings change because of the price effect. That is, the Delta Active Share gauges the incremental changes in Active Share driven by actual trading in quarter t.

4.3 Investment constraints and ownership variables

Almazan et al. (2004) discuss the six specific investment practices that are relevant to the operations of equity funds: (i) borrowing of money, (ii) margin purchases, (iii) short selling, (iv) writing or investing in options on equities, (v) writing or investing in stock index futures, and (vi) investments in restricted securities. Fund managers are required to disclose information about (i) whether specific investment policies are permitted and (ii) (if permitted) whether they engage in these investment practices during the reporting period by responding "yes" or "no" in Form N-SAR. The first three practices are related to leverage constraints, the fourth and fifth practices are related to derivatives constraints, and the last practice is related to illiquid assets constraints. Those practices will impose a binding constraint on managers' investment decisions because these variables affect the extent to which a fund deviates from its benchmark. To measure the extent of constraints that fund managers face, following Almazan et al. (2004), we compute an aggregate score to summarize a fund's overall constraint. The constraint score approach of Almazan et al. (2004) places an equal weight across the three distinct constraint categories. More precisely,

$$Investment\ Constraints = \frac{1}{3} * (\frac{1}{3} * total\ leverage\ constraints) + \frac{1}{3} * (\frac{1}{2} * total\ derivaties\ constraints) + \frac{1}{3} * (illiquid\ assets\ constraints). \tag{30}$$

The aggregate score varies between 0 and 1, and a higher score corresponds to a more constrained fund.[15] We also modify the original measure by focusing on the leverage constraint only, as this is a directly relevant constraint for altering the (Delta) Active Share.

The SEC required the disclosure of ownership starting in 2005. The disclosure is in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million) and is required for all portfolio managers of a fund. We obtain these data from Morningstar and created two variables: sum of maximum ownership and maximum ownership. The first ownership variable is the sum of the upper bound of each manager's ownership interval. The second ownership variable is the maximum of the upper bound of each manager's ownership interval. We have data on this variable from 2007 until the end of our sample in 2013.

4.4 The effects of overconfidence

4.4.1 Regression specification

To investigate the effect of overconfidence, following Choi and Lou (2010), we estimate the following regression model:

[15] See sections 2.2.1 and 2.2.2 of Almazan et al. (2004) for a more detailed explanation.

$$\begin{aligned}\Delta AS_{i,q,y} = \alpha + \beta_1 * SPR_{i,q-4:q-1} + \beta_2 * Experience_{i,y} \\ +\beta_3 * Experience_{i,y} * SPR_{i,q-4:q-1} + \beta_4 * Investment\,Constraints_{i,y} \\ +\beta_5 * Investment\,Constraints_{i,y} * SPR_{i,q-4:q-1} + \beta_6 * Ownership_{i,y} \\ +\beta_7 * Ownership_{i,y} * SPR_{i,q-4:q-1} + \gamma * Controls, \qquad (31)\end{aligned}$$

where the dependent variable ($\Delta AS_{i,q}$) is the change in the Active Share of fund i in quarter q that is purely attributable to the incremental trading activity. $\Delta AS_{i,q}$ is the difference between the Active Share constructed from the holdings reported at the end of quarter q and the hypothetical Active Share constructed from the portfolio if the manager simply carries forward the position from quarter q-1 to q. The most important independent variable $SPR_{i,q-4:q-1}$ is defined as the sum of only the positive excess return relative to the benchmark in the previous twelve months. The intuition behind this variable is that managers become overconfident because of the bias in their learning. They increase the precision of their signal when they get a positive feedback. However, they do not proportionately reduce their precision when their signals are not corroborated by future outcomes. The higher the sum of positive returns, the greater the magnitude of confirming signals.

We incorporate benchmark adjusted return from quarter q-4 to q-1 (*past return*) as a control variable. In addition, we include the tracking error and turnover in the concurrent period to reflect the investment styles. Cremers and Petajisto (2009) argue that tracking error and the Active Share are distinct active management measures in that one can choose tracking error as a proxy for factor bets and the Active Share for stock selection.[16] Other control variables are fund flows from q-4 to q-1, expense ratio, dummies for fund size, fund age, and fund styles at the end of quarter q-1. The regression specifications include quarter-fixed effects, and standard errors are clustered

[16]Our results are not sensitive to the exclusion of tracking error.

Table 3: Summary Statistics

This table provides the summary statistics for the sample of funds used in the paper. *Active Share* is defined as half of the sum of absolute deviations in portfolio weights of a fund from its benchmark index during the quarter q. *Delta Active Share* is defined as the Active Share of a fund at the end of quarter q minus the Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as the minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month total net assets of the fund. *Fund Age* is the number of years since the inception of a fund. *Investment Constrains* is an aggregate score to summarize a fund's overall investment constraint. The aggregate score varies between 0 and 1, and a higher score refers to a more constrained fund. Ownership in the fund is disclosed in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million) and is required for all portfolio managers of a fund. *Sum of Maximum Ownership* is the sum of the upper bound of each manager's ownership interval. *Maximum Ownership* is the maximum of the upper bounds of each manager's ownership interval.

	Mean	Std Dev	25th Pct	Median	75th Pct
Active Share	77%	16%	67%	79%	90%
Delta Active Share (%)	-3%	10%	-3%	-1%	0%
Sum of Positive Returns (*SPR*)	8%	8%	3%	6%	10%
Past Return (*12 months*) (%)	0%	8%	-4%	-1%	3%
Tracking Error (*12 months*) (%)	2%	1%	1%	1%	2%
Expense Ratio (%)	1.25%	0.43%	0.99%	1.20%	1.48%
Turnover (%)	89%	104%	36%	66%	110%
Fund Age	15.28	13	7	12	18
Investment Constraints(%)	22%	21%	11%	22%	28%
Sum of (*each*) *Maximum Ownership* ($)	0.65 mil	0.92 mil	0	0.27 mil	1 mil
Maximum Ownership ($)	0.39 mil	0.41 mil	0	0.1 mil	1 mil

at both the quarter and fund levels.

Table **3** provides the summary statistics of *Active Share*, *Delta Active Share*, and other fund characteristics used in the analyses. We are confident that the distribution of our *Active Share* measure is similar to that of Cremers and Petajisto (2009). Our average *Active Share* of about 80%, with a standard deviation of 16%, is very close to the summary statistic presented in Cremers and Petajisto (2009). On average, our *Delta Active Share* displays a mean (median) of -3% (-1%), which is comparable to the results of Choi and Lou (2010). However, a 10% standard deviation of change in *Active Share* clearly indicates a substantial variation among mutual funds. On average, the annual benchmark-adjusted return is close to 0 with a median of about

-100 basis points. Our main independent variable of interest, *SPR*, has a mean of 8% and a standard deviation of 8%. The 6% median indicates that the distribution of this variable is slightly right skewed. Finally, the mean investment restrictions proxied by the aggregate constraint score is 0.22, and portfolio managers, on average, own about 0.39 to 0.65 million dollars in the fund, depending on the definition of ownership.

4.4.2 Empirical results

We estimate a pooled OLS regression, and the unit of our analysis is at the fund-quarter level. Table 4 presents our first regression results. We first replicate the baseline results of Choi and Lou (2010) to test for the existence of self-attribution bias among fund managers. Consistent with their findings, $SPR_{i,q-4:q-1}$ is positive and statistically significant after including a host of control variables. Importantly, the benchmark-adjusted return over the past four quarters (*past return)* has no effect on the ΔAS. This further illustrates that managers do not decrease the perceived precision of their private signals upon receiving disconfirming feedback.

Managerial experience plays a crucial role in learning. With time, managers learn about their own ability and exhibit lower bias (see Gervais and Odean (2001)). We use the number of years since fund inception as our measure of the manager's experience. We define a new dummy variable, $Experience\,Proxy$, which takes a value of one if the manager has above the median experience in that quarter. Column 2 of Table 4 displays the effect of managerial experience on the attribution bias. Consistent with prior literature, more experienced managers show significantly lower bias.

Table 4: Overconfidence and Fund Investment Constraints

This table presents the regression of *Delta Active Share* on *SPR*, *Past Return*, and other fund characterstics. Regression results include an interaction term of *SPR* with (i) *Experience Proxy* and (ii) *Investment Constraints*, respectively. We use the number of years since the fund's inception as a proxy for the manager's experience. *Experience Proxy* takes a value of one if the manager has above median experience in that quarter. *Investment Constraints* is an aggregate score to summarize a fund's overall investment constraint. The aggregate score varies between 0 and 1, and a higher score refers to a more constrained fund. In addition to a constraint score measure (*Investment Constraints*) specified in *Eq.* (30), we modify the original measure by focusing on the leverage constraint only (*Narrow Constraints*). *Investment Constraints* takes a value of one if the manager has above median constraints in that quarter. *Delta Active Share* is defined as the Active Share of a fund at the end of quarter q minus the Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as the minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month total net assets of the fund. *Fund Age* is the number of years since the inception of a fund. *Style Code* 1-4 are investment objective code dummies (2 = aggressive growth, 3 = growth, 4 = growth and income). *Age Categories* 1-4 are the age (number of years since inception) quartile dummies (1 = youngest). *Size Categories* 1-4 are the fund size quartile dummies (1 = largest). The subscript q represents quarter q and the timing of the variables. Heteroskedasticity-consistent t-statistics are in parentheses. The significance levels are denoted by *, **, and *** and, indicate whether the results are statistically different from zero at the 10%, 5%, and 1% significance levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$SPR_{q-4:q-1}$ (%)	0.150***	0.176***	0.125**	0.155**	0.133***	0.163**
	(0.055)	(0.061)	(0.048)	(0.063)	(0.049)	(0.065)
Experience Proxy		0.002		0.003		0.003
		(0.005)		(0.005)		(0.005)
$SPR_{q-4:q-1}$**Experience Proxy*		-0.047**		-0.056***		-0.055***
		(0.020)		(0.018)		(0.018)
Investment Constrains			0.008***	0.008***		
			(0.002)	(0.002)		
$SPR_{q-4:q-1}$**Investment Constrains*			-0.029*	-0.030*		
			(0.015)	(0.015)		
Narrow Constraints					0.005***	0.006***
					(0.002)	(0.002)
$SPR_{q-4:q-1}$**Narrow Constraints*					-0.028**	-0.031**
					(0.013)	(0.015)
Tracking Error$_{q-4:q-1}$(%)	-0.111	-0.016	-0.035	0.039	-0.034	0.039
	(0.224)	(0.241)	(0.200)	(0.258)	(0.200)	(0.257)
Past Return$_{q-4:q-1}$(%)	-0.032	-0.037	-0.016	-0.019	-0.017	-0.020
	(0.027)	(0.032)	(0.024)	(0.030)	(0.024)	(0.030)
Expense Ratio$_{q-4:q-1}$ (%)	1.102***	1.192***	0.555***	0.554**	0.576***	0.579**
	(0.329)	(0.390)	(0.207)	(0.236)	(0.208)	(0.237)
Turnover Ratio$_{q-4:q-1}$ (%)	-0.010***	-0.012***	-0.011***	-0.013***	-0.011***	-0.013***
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Fund Flows$_{q-4:q-1}$ (%)	0.000***	0.000***	0.000***	0.000***	0.000***	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Style 1	0.002	0.002	-0.000	-0.000	-0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Style 2	0.001	0.001	-0.001	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
Style 3	0.009***	0.007**	0.003	0.002	0.003	0.001
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
Size 1	-0.002	-0.000	-0.002	-0.001	-0.003	-0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Size 2	-0.002	0.000	-0.001	0.000	-0.002	-0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Size 3	-0.003*	-0.001	-0.000	0.001	-0.001	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Age Category 1	0.003	0.002	0.001	0.001	0.002	0.001
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)
Age Category 2	0.005**	0.005	0.004*	0.004	0.004**	0.004
	(0.002)	(0.004)	(0.002)	(0.005)	(0.002)	(0.005)
Age Category 3	0.005*	0.004	0.001	0.000	0.002	0.001
	(0.003)	(0.005)	(0.002)	(0.005)	(0.002)	(0.005)
Observations	73,372	56,370	54,971	44,916	54,971	44,916
Adjusted R-squared	0.023	0.030	0.029	0.036	0.028	0.035
Fixed Effect	Qtr	Qtr	Qtr	Qtr	Qtr	Qtr
Clustering	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund

The main variable of our focus is *Investment Constraints* and its effect on *SPR*. The computed portfolio constraints variable is a continuous variable between 0 and 1. To make sense of the interaction coefficient, we convert it into a dummy variable that takes a value of one if the manager has above the median constraints in that quarter. Column 3 presents our main findings. Contrary to reasonable expectations, having additional investment constraints does not directly reduce the magnitude of portfolio deviations. However, the coefficient of interaction between *Investment Constraints* and *SPR* is negative and statistically significant, providing support for hypothesis 1. In addition, focusing on leverage constraints makes more sense because the other two constraints are not likely to be related to the deviation from benchmark holdings. Therefore, we also interact SPR with $Narrow\,Constraints$, a dummy variable that takes a value of one if the manager has above the median "*leverage*" constraints in that quarter. A strong negative result continues to emerge when we use a narrower measure of investment constraints. It is evident that the presence of portfolio constraints significantly inhibits the attribution bias the managers display, and hence overconfidence. We also acknowledge the endogeneity of the portfolio constraints that managers face. Nevertheless, our results show an association between investment constraints and the bias in the manager's action as we analyze managerial behavior in the quarter after observing the constraints. Importantly, after controlling for the direct effect, $Investment\,Constraints$ diminishes the effect of SPR on ΔAS. The magnitude of the two-way interaction term is about 4 times the size of the direct effect.

We now focus on the manager's variable compensation and its effect on attribution bias. Information on managerial ownership, our proxy for variable compensation, is presented as a range. We use the upper bound of these intervals and convert them into dollar values. Also, we convert the ownership variables into dummy variables that take a value of one if they are above the median in that quarter. Results for two different measures of ownership are displayed in Table 5. Consistent with hypothesis 2, if managers display attribution bias, this effect is substantially diminished in the presence of managerial ownership. We do not make a causal claim here. Clearly, the ownership in the fund depends on the personal portfolio decision of the fund manager. However, fund families often require managers to invest in their own fund(s) (see Laise (2006)).

Overall, we find that the empirical results are consistent with the predictions of our model. Portfolio constraints and compensation parameters can be, and often are, designed to mitigate some of the known agency problems and induce higher effort when hiring an overconfident manager.

Table 5: Overconfidence and Manager Ownership

This table presents the regression of *Delta Active Share* on *SPR*, *Past Return*, and other fund characterstics. Regression results include an interaction term of *SPR* with (i) the manager's ownership in the fund and (ii) *Experience Proxy*, respectively. *Delta Active Share* is defined as the Active Share of a fund at the end of quarter q minus the Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as the minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month total net assets of the fund. *Fund Age* is the number of years since the inception of a fund. Ownership in the fund is disclosed in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million) and is required for all portfolio managers of a fund. *Sum of Maximum Ownership* is the sum of the upper bound of each manager's ownership interval. *Maximum Ownership* is the maximum of the upper bounds of each manager's ownership interval. Each ownership variable takes a value of one if the manager has above median ownership in the fund that quarter. *Style Code* 1-4 are investment objective code dummies (2 = aggressive growth, 3 = growth, 4 = growth and income). *Age Categories* 1-4 are the age (number of years since inception) quartile dummies (1 = youngest). *Size Categories* 1-4 are the fund size quartile dummies (1 = largest). The subscript q represents quarter q and the timing of the variables. Heteroskedasticity-consistent t-statistics are in parentheses. The significance levels are denoted by *, **, and *** and, indicate whether the results are statistically different from zero at the 10%, 5%, and 1% significance levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)
$SPR_{q-4:q-1}$ (%)	0.291**	0.301**	0.333**	0.341**
	(0.121)	(0.125)	(0.120)	(0.124)
Sum of Maximum Ownership	0.018***		0.018***	
	(0.004)		(0.004)	
$SPR_{q-4:q-1}$ (%) * *Sum of Maximum Ownership*	-0.131***		-0.133***	
	(0.043)		(0.044)	
Maximum Ownership		0.020***		0.020***
		(0.005)		(0.005)
$SPR_{q-4:q-1}$ (%) * *Maximum Ownership*		-0.142***		-0.145***
		(0.050)		(0.050)
Experience Proxy			0.008	0.007
			(0.005)	(0.005)
$SPR_{q-4:q-1}$ (%) * *Experience Proxy*			-0.050	-0.045
			(0.036)	(0.036)
$Tracking\ Error_{q-4:q-1}$(%)	-0.081	-0.066	-0.137	-0.120
	(0.406)	(0.404)	(0.402)	(0.400)
$Past\ Return_{q-4:q-1}$(%)	-0.072	-0.070	-0.077	-0.074
	(0.047)	(0.046)	(0.047)	(0.046)
$Expense\ Ratio_{q-4:q-1}$ (%)	0.816*	0.819**	0.802*	0.808*
	(0.398)	(0.397)	(0.413)	(0.412)
$Turnover\ Ratio_{q-4:q-1}$ (%)	-0.014***	-0.014***	-0.014***	-0.014***
	(0.003)	(0.003)	(0.003)	(0.003)

VARIABLES	(1)	(2)	(3)	(4)
$Fund\,Flows_{q-4:q-1}$ (%)	0.000**	0.000**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)
$Style$ 1	0.002	0.002	0.002	0.002
	(0.003)	(0.003)	(0.003)	(0.003)
$Style$ 2	0.001	0.001	0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.003)
$Style$ 3	0.005	0.004	0.004	0.004
	(0.003)	(0.003)	(0.003)	(0.003)
$Size$ 1	-0.001	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.003)	(0.003)
$Size$ 2	0.001	0.002	0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.003)
$Size$ 3	-0.001	-0.000	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)
$Age\,Category$ 1	-0.001	-0.001	-0.000	-0.000
	(0.004)	(0.004)	(0.004)	(0.004)
$Age\,Category$ 2	0.001	0.001	-0.002	-0.003
	(0.003)	(0.003)	(0.006)	(0.006)
$Age\,Category$ 3	-0.002	-0.002	-0.006	-0.006
	(0.004)	(0.004)	(0.005)	(0.005)
Observations	28,152	28,152	27,724	27,724
Adjusted R-squared	0.043	0.044	0.047	0.048
Fixed Effect	Qtr	Qtr	Qtr	Qtr
Clustering	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund

5 Conclusion

It is well established that individuals are overconfident. Barring a few exceptions, most papers ignore this trait in designing compensation contracts. Similarly, when studying behavioral biases, it is imperative that we include the agent's incentives in the analysis.

Here, we study the problem of a principal who wishes to delegate portfolio management to an overconfident agent and has to choose an appropriate compensation contract. In this framework, we find that managerial effort as a function of overconfidence increases up to a threshold. Thereafter, it decrease on account of restrictions on the agent's portfolio choices. The investor can gain from commitment to such high effort because this increases the conditional expected return of the portfolio. However, additional effort also leads to incremental risk taking. By designing appropriate incentives, the investor can reduce the level of portfolio risk to some optimal level. These gains are not unbounded because the costs outweigh the benefits beyond a certain threshold. Overall, it is not surprising that we find evidence of overconfidence in fund management. A moderate amount of it seems to be optimal.

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A Proofs

A.1 Optimal level of risky assets

The optimal quantity chosen is the solution to the following maximization problem

$$\max_\theta E_m(U(W_m)) = \max_\theta E_m(-\exp\{-aF - a\theta\tilde{x}|y\}),$$

where $\tilde{x}|y$ is the return distribution conditional on observing the signal y. Given the distribution of $\tilde{x}|y$

$$E_m(U(W_m)) = -\exp\{-aF\}\exp\left\{-a\alpha\theta y\tfrac{e\psi}{1+e\psi} + \tfrac{1}{2}(a\alpha\theta)^2\tfrac{1}{1+\psi e}\right\}$$

and the first order condition for the quantity demanded θ is going to be

$$a\alpha\theta\tfrac{1}{1+\psi e} - a\alpha y\tfrac{e\psi}{1+e\psi} = 0.$$

This implies that the optimal level of risky assets in the portfolio is given by

$$\theta = \frac{e\psi}{a\alpha}y. \tag{a.1}$$

A.2 Expected Utility of the Manager

Knowing the quantity demanded by the manager, his expected utility given the level of effort and the signal is given by

$$E_m(U|y) = -E[\exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}]$$

$$= -\int_{-\infty}^{\infty} \exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}f(x|y)dx.$$

$f(x|y)$ is the conditional return distribution. The above integral is over all the states that are possible after the portfolio has been picked. Simplifying the expression further we have

$$E_m(U|y) = -\int_{-\infty}^{\infty} \exp\left\{-aF - a\alpha\frac{e\psi}{a\alpha}y\tilde{x}|y + V(a,e)\right\} f(x)dx$$

$$= -\exp\{-aF + V(a,e)\}\int_{-\infty}^{\infty} \exp\{-e\psi y\tilde{x}|y\} f(x)dx$$

$$= -\exp\{-aF + V(a,e)\}\exp\left\{-e\psi y\frac{e\psi}{1+e\psi}y + \frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}$$

$$E_m(U|y) = -\exp\{-aF + V(a,e)\}\exp\left\{-\frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}.$$

The unconditional expected utility of the manager, which is the integral of above with respect to all the possible signals is going to be

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}E\left[\exp\left\{-\frac{1}{2}\frac{(e\psi\tilde{y})^2}{(1+e\psi)}\right\}\right]$$

$$= -\exp\{-aF+V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e\psi}{1+e\psi}\frac{y^2}{2}\right\} f(y)$$

$$= -\exp\{-aF+V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{e\psi}{(1+e\psi)}(y^2 e\psi + y^2)\right\} f(y)$$

$$= -\exp\{-aF + V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}e\psi y^2\right\}.$$

For a random variable which is distributed $N\left(0, \frac{1}{e\psi}\right)$ the following is true

$$\int_{-\infty}^{\infty} f(y)dy = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\frac{2\pi}{e\psi}}} \exp\left\{-\frac{1}{2}e\psi y^2\right\} = 2 * \int_{0}^{\infty} \frac{1}{\sqrt{\frac{2\pi}{e\psi}}} \exp\left\{-\frac{1}{2}e\psi y^2\right\} dy.$$

Using the above expression we have

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}2 * \sqrt{\frac{1}{1+e\psi}} \int_{0}^{\infty} \frac{1}{\sqrt{\frac{2\pi}{e\psi}}} \exp\left\{-\frac{1}{2}e\psi y^2\right\} dy.$$

At this point substitute $s = e\psi y^2$. This substitution will give us $\frac{1}{2e\psi y}ds = dy$. Also if $y = 0$ then $s = 0$ and if $y = \infty$ then $s = \infty$. Since $y = \sqrt{\frac{s}{e\psi}}$ we have

$$E_m(U|e) = -\exp\{-aF+V(a,e)\}2*\sqrt{\frac{1}{1+e\psi}} \int_{0}^{\infty} \frac{1}{\sqrt{\frac{2\pi}{e\psi}}} \exp\left\{-\frac{s}{2}\right\} \frac{1}{2e\psi\sqrt{\frac{s}{e\psi}}}ds.$$

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}\sqrt{\frac{1}{1+e\psi}} \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{s}{2}\right\} s^{\frac{-1}{2}} ds.$$

The term in the integral is the distribution function of the $\chi^2(1)$ random variable so $\int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{s}{2}\right\} s^{\frac{-1}{2}} ds \to 1$. Therefore the unconditinal expectation of the portfolio manager is given

$$E_m(U|e) = -\exp\{-aF + V(a,e)\} \left(\frac{1}{1+e\psi}\right)^{1/2}. \tag{a.2}$$

A.3 Proof to Proposition 1

Equation (10), the first order condition for effort, can be written as the following

$$V'(a,e_{fb}) - \frac{\psi}{2}(\frac{1}{1+\psi e_{fb}}) = 0$$

Let the function F be

$$F = V'(a,e_{fb}) - \frac{\psi}{2}(\frac{1}{1+\psi e_{fb}})$$

Based on the implicit function theorem we have that

$$\frac{\partial e}{\partial \psi} = -\frac{\frac{\partial F}{\partial \psi}}{\frac{\partial F}{\partial e}}$$

By definition of optimality we know that $\frac{\partial F}{\partial e_{fb}} < 0$. So, in order to prove the proposition need to show that $\frac{\partial F}{\partial \psi} > 0$. Differentiating F with respect to ψ we have

$$\frac{\partial F}{\partial \psi} = \frac{1}{2}\left(\frac{1}{1+e\psi}\right) - \frac{\psi e}{2}\left(\frac{1}{(1+e\psi)^2}\right) = \frac{1}{2}\left(\frac{1}{(1+e\psi)^2}\right) > 0.$$

Therefore we are done.

A.4 Investor's Expected Utility function

As mentioned in the main text,we are going to assume that the investor has the same preferences as the manager; including the level of risk aversion. Using the investor's conditional terminal wealth given in equation (3) the conditional expected utility of the investor is going to be

$$E_i(U|y) = -E\left[\exp\left\{-a(1-\alpha)\frac{e\psi}{a\alpha}y\tilde{x}|y + aF\right\}\right]$$

$$= -\exp\{aF\}\int_{-\infty}^{\infty}\exp\left\{-a(1-\alpha)\frac{e\psi}{a\alpha}y\tilde{x}|y\right\}f(x|y)dx$$

$$= -\exp\{aF\}\exp\left\{-\frac{(1-\alpha)e\psi}{\alpha}\frac{e}{1+e}y^2 + \frac{1}{2}\left(\frac{(1-\alpha)e\psi}{\alpha}\right)^2\frac{y^2}{1+e}\right\}$$

$$= -\exp\{aF\}\exp\left\{-\frac{(1-\alpha)\psi}{\alpha}\frac{e^2}{1+e}y^2\left(1-\frac{1}{2}\left(\frac{(1-\alpha)\psi}{\alpha}\right)\right)\right\}.$$

Define two new variables

$$m(\alpha) = \left(\frac{1-\alpha}{\alpha}\right)\psi, and$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)).$$

Substituting these variables in the above equation we have

$$E_i(U|y) = -\exp\{aF\}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha)\right\}. \tag{a.3}$$

We can now compute the unconditional expected utility of the investor by integrating over the range of possible signals y

$$E_i(U) = -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha)\right\}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e}{1+e}\frac{y^2}{2}\right\}dy$$

$$= -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha) - \frac{e}{1+e}\frac{y^2}{2}\right\}dy$$

$$= -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{ey^2}{2(1+e)}(eM(\alpha)+1)\right\}dy.$$

Substitute $s = \frac{ey^2}{2(1+e)}(eM(\alpha)+1)$ in the above equation. Simplifying it further leads to the following expression for the investor's unconditional expected utility

$$E_i(U) = -\exp\{aF\}\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}. \tag{a.4}$$

A.5 Proof to Lemma 1

In order to compute the contract parameters the investor has to solve a constrained optimization problem where the constraint is on participation given by (13). The Lagrangian of the problem is the following

$$\mathcal{L} = -e^{aF}\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{\frac{1}{2}} + \lambda\left(-\exp\{-aF+V(a,e_{fb})\}\left(\frac{1}{1+e_{fb}\psi}\right)^{\frac{1}{2}} + U_0\right).$$

Notice that the participation constraint is not a function of α. We have the following first order condition with respect to α

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -\exp\{aF\}(\tfrac{-1}{2})\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{3/2} e_{fb}M'(\alpha,\psi){=}0.$$

Look in the proof of expected utility of the investor for definitions of $m(\alpha)$ and $M(\alpha)$. The above condition is equivalent to

$$\frac{\partial M(\alpha,\psi)}{\partial \alpha} = \frac{2\psi}{\alpha^2}(m(\alpha)-1) = 0.$$

Solving the above equation for α

$$\alpha_{\text{fb}} = \frac{\psi}{1+\psi}.$$

The other first order condition is with respect to F $(\frac{\partial \mathcal{L}}{\partial F})$ and is given by

$$-ae^{aF}\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{1/2} + \lambda\left(ae^{-aF+V(a,e_{fb})}\left(\frac{1}{1+e_{fb}\psi}\right)^{1/2}\right) = 0$$

$$\left(\frac{1+e_{fb}\psi}{1+e_{fb}M(\alpha,\psi)}\right)^{1/2} = \lambda e^{-2aF+V(a,e_{fb})}$$

taking the log of both sides we have

$$\tfrac{1}{2}log\left(\tfrac{1+e_{fb}\psi}{1+e_{fb}M(\alpha,\psi)}\right) = \log(\lambda) - 2aF + V(a, e_{fb}).$$

Notice that at the point of optimality

$$M(\alpha_{\text{fb}}) = \tfrac{1}{\psi}\psi\left(2 - \tfrac{1}{\psi}\psi\right) = 1.$$

Substituting this is previous equation and solving for F will gives us

$$F = \frac{1}{2a}\left[\log(\lambda) + V(a, e_{fb}) - \frac{1}{2}\log\left(\frac{1+e_{fb}\psi}{1+e_{fb}}\right)\right]. \tag{a.5}$$

However, this is still an unknown function of λ. Since the investor does not gain from paying anything more than the reservation utility, the participation constraint will be binding at the optimum. So the following equality should hold

$$\exp\{-aF^{*} + V(a, e_{fb})\}\left(\tfrac{1}{1+e_{fb}\psi}\right)^{1/2} = U_0.$$

Expanding this further and taking the log of both sides we get

$$\log(\lambda) = \frac{1}{2}\log\left(\frac{1+e_{fb}\psi}{1+e^{*}}\right) + V(a, e_{fb}) + \log\left(\frac{1}{1+e_{fb}\psi}\right) - 2\log(U_o). \tag{a.6}$$

Substituting (a.6) in (a.5) we get

$$F = \left[\tfrac{1}{a}V(a, e_{fb}) + \tfrac{1}{2a}\log\left(\tfrac{1}{1+e_{fb}\psi}\right) - \tfrac{1}{a}\log(U_o)\right].$$

A.6 Proof to Proposition 2

From Lemma 1 we know the optimal contract parameters. Substituting them in the expected utility function of the investor, equation (12), we get the following

$$E_i(U) = -e^{[V(a,e_{fb}) - \log(U_o) + \frac{1}{2}\log\left(\frac{1}{1+e_{fb}\psi}\right)]}\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{1/2}.$$

In order to determine if this function is increasing in overconfidence, differentiate the above equation with respect to ψ. Below is the expression

$$\frac{\partial}{\partial\psi}\left[-e^{V(a,e^*)+\frac{1}{2}\log\left(\frac{1}{1+e^*\psi}\right)}\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}\right]$$

$$= -e^{V(a,e^*)+\log\left(\frac{1}{1+e^*\psi}\right)^{\frac{1}{2}}}\frac{\partial}{\partial\psi}\left(\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}\right) -$$
$$\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}\frac{\partial}{\partial\psi}\left[e^{V(a,e^*)+\log\left(\frac{1}{1+e^*\psi}\right)^{\frac{1}{2}}}\right].$$

Remember, $M(\alpha_{\text{fb}}) = 1$. Lets focus on

$$\frac{\partial}{\partial\psi}\left[e^{V(a,e^*)+\frac{1}{2}\log\left(\frac{1}{1+e^*\psi}\right)}\right] = \frac{\partial}{\partial\psi}\left[e^{V(a,e^*)}\left(\frac{1}{1+e^*\psi}\right)^{\frac{1}{2}}\right]$$

$$= \left[\left(\frac{1}{1+e^*\psi}\right)^{\frac{1}{2}} e^{V(a,e^*)}\frac{\partial V(a,e^*)}{\partial e^*}\frac{\partial e^*}{\partial\psi} + e^{V(a,e^*)}\left(-\frac{1}{2}\right)\left(\frac{1}{1+e^*\psi}\right)^{\frac{3}{2}}\left(\frac{\partial e^*}{\partial\psi}\psi + e^*\right)\right]$$

$$= \left[\left(\frac{1}{1+e^*\psi}\right)^{\frac{1}{2}} e^{V(a,e^*)}\frac{\partial e^*}{\partial\psi}\left(\frac{\partial V(a,e^*)}{\partial e^*} - \frac{\Psi}{2}\left(\frac{1}{1+e^*\psi}\right)\right) - e^{V(a,e^*)}\frac{e^*}{2}\left(\frac{1}{1+e^*\psi}\right)^{\frac{3}{2}}\right].$$

Note, that the first order condition for effort

$$\left(\frac{\partial V(a,e^*)}{\partial e^*} - \frac{\Psi}{2}\left(\frac{1}{1+e^*\psi}\right)\right) = 0.$$

Therefore,

$$\frac{\partial}{\partial\psi}\left[e^{V(a,e^*)+\frac{1}{2}\log\left(\frac{1}{1+e^*\psi}\right)}\right] = -e^{V(a,e^*)}\frac{e^*}{2}\left(\frac{1}{1+e^*\psi}\right)^{\frac{3}{2}} < 0.$$

Also, since we already know that effort is increasing in overconfidence, it has to be that $\frac{\partial}{\partial\psi}\left[\left(\frac{1}{1+e^*}\right)^{1/2}\right] < 0$. Therefore $\frac{\partial E_I(U)}{\partial\psi} > 0\ \forall\psi$.

A.7 Proof to Proposition 3

From Lemma 1 we already know that $\alpha_{fb} = \frac{\psi}{1+\psi}$. We also know that the quantity of risky asset demanded by the manager is given by $\theta(y) = \frac{e\psi}{a\alpha} y$. Substituting the value of α_{fb} in the demand function we get that

$$\theta(y) = \frac{ey}{a}(1+\psi).$$

From the above equation it is clear that the equilibrium risky quantity is increasing in ψ.

A.8 Proof to Lemma 2

Given the effort level and the signal observed the expected utility of the manager hinges on the quantity demanded. We have the following expression for the utility function

$$E_m(U|y) = -E[\exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}]$$

$$= -e^{-aF+V(a,e)} \int_{-\infty}^{\infty} \exp\{-a\alpha\theta(y)\tilde{x}|y\} f(x|y)dx.$$

For signals below the bound ($y < \frac{-ka\alpha}{\psi e}$)

$$\int_{-\infty}^{\infty} \exp\{a\alpha k\tilde{x}|y\} f(x|y)dx = \exp\left\{\frac{\psi e(a\alpha ky)}{1+\psi e} + \frac{(ka\alpha)^2}{2}\frac{1}{1+\psi e}\right\}$$

$$= \exp\left\{\frac{\psi e}{1+\psi e} ka\alpha\left(y + \frac{(ka\alpha)}{2\psi e}\right)\right\}. \tag{a.7}$$

For signals above the bound ($y > \frac{ka\alpha}{\psi e}$)

$$\int_{-\infty}^{\infty} \exp\{-a\alpha k\tilde{x}|y\} f(x|y)dx = \exp\left\{\frac{-\psi e(a\alpha k y)}{1+\psi e} + \frac{(ka\alpha)^2}{2}\frac{1}{1+\psi e}\right\}$$

$$= \exp\left\{\frac{-\psi e}{1+\psi e} ka\alpha \left(y - \frac{(ka\alpha)}{2\psi e}\right)\right\}. \qquad \text{(a.8)}$$

<u>*For signals within the bound*</u> ($|y| \leq \frac{ka\alpha}{\psi e}$)

$$\int_{-\infty}^{\infty} \exp\left\{-a\alpha \frac{e\psi}{a\alpha} y\tilde{x}|y\right\} f(x|y)dx = \exp\left\{\frac{-(\psi e y)^2}{1+\psi e} + \frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}\right\} = \exp\left\{-\frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}\right\}.$$

Now lets integrate over all possible signals and solve for the unconditional expected utility of the manager. We still have to deal with the three regions separately. For the signals below the threshold we get the following expression as the share towards expected utility

$$-e^{-aF+V(a,e)} \int_{-\infty}^{-\frac{ka\alpha}{\psi e}} e^{\left\{\frac{\psi e}{1+\psi e} ka\alpha \left(y + \frac{(ka\alpha)}{2\psi e}\right)\right\}} f(y)\, dy$$

where $f(y)$ is the distribution function of a random variable which is distributed $N\left(0, \frac{1+\psi e}{\psi e}\right)$. Applying the normal distribution's density function to the above equation we get

$$= -e^{-aF+V(a,e)} \int_{-\infty}^{-\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\frac{-y^2}{2}\left(\frac{\psi e}{1+\psi e}\right)} e^{\left\{\frac{\psi e}{1+\psi e} ka\alpha \left(y + \frac{(ka\alpha)}{2\psi e}\right)\right\}} dy.$$

Using completion of squares we have

$$= -e^{-aF+V(a,e)} \int_{-\infty}^{-\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\left\{\frac{-e\psi}{2(1+e\psi)}(y-ka\alpha)^2\right\}} e^{\frac{(ka\alpha)^2}{2}} dy.$$

Going to make a substitution $s = \frac{e\psi}{1+e\psi}(y - ka\alpha)^2$. It can be proved that after this substitution the share of expected utility from the signal below the threshold is given by

$$= -\frac{1}{2}(e^{-aF+V(a,e)})\left(e^{\frac{(ka\alpha)^2}{2}}\right) \int_{\frac{(ka\alpha)^2}{\psi e}(1+\psi e)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\left\{-\frac{s}{2}\right\}} \frac{1}{\sqrt{s}} ds.$$

The function in the integral is the probability density function of a $\chi^2(1)$ random variable. Let ϕ and Φ be the density density and the cumulative distribution function of $\chi^2(1)$ random variable. Also, one would get an exact same equation for the part above the threshold. For brevity, we don't show that proof here. Now, for the contribution of the part of the signal space which is within the bounds ($|y| \leq \frac{ka\alpha}{\psi e}$). The expression below represents that part of the expected utility.

$$-e^{-aF+V(a,e)} \int_{-\frac{ka\alpha}{\psi e}}^{\frac{ka\alpha}{\psi e}} \exp\left\{-\frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}\right\} f(y)\, dy$$

$$= -e^{-aF+V(a,e)} \int_{-\frac{ka\alpha}{\psi e}}^{\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\frac{-y^2}{2}\left(\frac{\psi e}{1+\psi e}\right) - \frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}} dy.$$

Substituting $s = \psi e y^2$ we get the following

$$= -e^{-aF+V(a,e)}\left(\frac{1}{1+\psi e}\right)^{1/2}\int\limits_{0}^{\frac{(ka\alpha)^2}{\psi e}}\frac{1}{\sqrt{2\pi}}e^{\frac{-s^2}{2}}\frac{1}{\sqrt{s}}ds.$$

Adding the three parts we get the following expression as the unconditional expected utility of the manager

$$E_m(U) = -e^{-aF+V(a,e)}\left[\left(\frac{1}{1+\psi e}\right)^{\frac{1}{2}}\Phi\left(\frac{(ka\alpha)^2}{\psi e}\right)\right]+$$
$$- e^{-aF+V(a,e)}\left[exp\left(\frac{(ka\alpha)^2}{2}\right)\left(1-\Phi\left(\frac{(ka\alpha)^2}{\psi e}(1+\psi e)\right)\right)\right].$$

A.9 Proof to Lemma 3

Credit for the proof goes to Gómez and Sharma (2006). The result almost follows from the Lemma 1 and Corollary 2 in their paper. Equation (18) describes the first order condition for effort. Lets define a function M as follows

$$M := V'(a,e^*)g(e,\psi|\alpha) + g'(e^*,\psi|\alpha).$$

Then, using M and the implicit function theorem the proof would be complete if we can show that $\frac{\partial M}{\partial \alpha} < 0$. This is true because by definition, $\frac{\partial M}{\partial e^*} > 0$. Further, using Lemma 1 in Gómez and Sharma (2006) $\frac{\partial g(e^*,\psi|\alpha)}{\partial \alpha} < 0$. Also, from the definition of $g'(e^*,\psi|\alpha)$ in equation (19), we can see that $\frac{\partial g'(e^*,\psi|\alpha)}{\partial \alpha} < 0$. Moreover, by assumption the effort function, $V(a,e)$, is convex and increasing function for all levels of effort therefore $V'(a,e^*) > 0$. Using these facts, $\frac{\partial M}{\partial \alpha} = V'(a,e^*)\frac{\partial g(e^*,\psi|\alpha)}{\partial \alpha} + \frac{\partial g'(e^*,\psi|\alpha)}{\partial \alpha} < 0$.

This concludes the proof. On a related note, the proofs relating to the existence of a unique optimal second best effort, the continuity of the effort function with respect to α, and the differentiability of the effort function with respect to α are all applicable to the model here just as they were in Gómez and Sharma (2006).

A.10 Proof to Lemma 4

Note, the principal in this case is a rational person. Therefore, in evaluating investor's expected utility rational beliefs should be used. Expected utility of the investor given the effort level and the signal is

$$E_i\left(U\left|y\right.,e\right)=-E\left[\exp\left(-a\left(1-\alpha\right)\theta\tilde{x}\left|y+aF\right.\right)\right].$$

But the θ is dependent on the signal and on account of the constraints on holdings, like the proof of Lemma 2, there are three distinct cases to deal with.

Conditional Expectation

For signals within the bound $(|y|\leq\frac{ka\alpha}{\psi e})$

$$E_i\left(U\left|y,e\right.\right)=-E\left[\exp\left(-a\left(1-\alpha\right)\tfrac{e\psi}{a\alpha}y\tilde{x}\left|y+aF\right.\right)\right]$$

$$=-\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)\tfrac{e\psi}{a\alpha}y\tilde{x}\left|y\right.\right)\right].$$

Knowing the distribution of the $\tilde{x}\left|y\right.$, the above expectation can be written as following

$$=-\exp\left(aF\right)\left[\exp\left(-\tfrac{(1-\alpha)}{\alpha}\tfrac{e^2y^2\psi}{1+e}+\tfrac{1}{2}\left(\tfrac{(1-\alpha)}{\alpha}\right)^2\tfrac{e^2\psi^2y^2}{1+e}\right]\right.$$

Simplifying this further we have

$$=-\exp\left(aF\right)\left[\exp\left(-\tfrac{(1-\alpha)}{\alpha}\psi\tfrac{e^2y^2}{1+e}\left(1-\tfrac{1}{2}\left(\tfrac{(1-\alpha)}{\alpha}\right)\psi\right)\right]\right.$$

Like before, let us assume $m\left(\alpha\right)=\psi\frac{(1-\alpha)}{\alpha}$ and $M\left(\alpha\right)=m\left(a\right)\left(2-m\left(a\right)\right).$ Then,

$$E_i\left(U\left|y,e\right.\right) = -\exp\left(aF\right)\exp\left(-\tfrac{1}{2}\tfrac{e^2y^2}{1+e}M\left(\alpha\right)\right).$$

<u>*For signals below the bound*</u> $(y < \frac{-ka\alpha}{\psi e})$

$$E_i\left(U\left|y,e\right.\right) = -\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)\left(-k\right)\tilde{x}\left|y\right.\right)\right].$$

Evaluating the expectation we have the following

$$E_i\left(U\left|y,e\right.\right) = -\exp\left(aF\right)\exp\left(\tfrac{ak(1-\alpha)}{1+e}\left(ye+\tfrac{1}{2}ak\left(1-\alpha\right)\right)\right).$$

<u>*For signals above the bound*</u> $(y > \frac{ka\alpha}{\psi e})$

$$E_i\left(U\left|y,e\right.\right) = -\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)k\tilde{x}\left|y\right.\right)\right].$$

Evaluating the expectation we get the following expression

$$E_i\left(U\left|y,e\right.\right) = -\exp\left(aF\right)\exp\left(\tfrac{-ak(1-\alpha)}{1+e}\left(ye-\tfrac{1}{2}ak\left(1-\alpha\right)\right)\right).$$

<u>Unconditional Expectaion</u>

Using the above computed conditional expected utility, now we are going to compute the unconditional expected utility, which is taking the expectation over all possible signals. Like before, there are going to be three different regions over which we need to integrate.

<u>*For signals within the bound*</u> $(|y| \leq \frac{ka\alpha}{\psi e})$

$$E_i\left(U\left|e\right.\right) = -\exp\left(aF\right)\int_{-\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\frac{\mathrm{ka}\,\alpha}{\psi e}}\exp\left(-\tfrac{1}{2}\tfrac{e^2y^2}{1+e}M\left(\alpha\right)\right)f\left(y\right)dy,$$

where $f\left(y\right)$ is the density function of the normal distribution given as $N\left(0, \frac{1+e}{e}\right)$. Using the distribution function of the gaussian random variable we get

$$= -\exp\left(aF\right)\int_{-\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\frac{\mathrm{ka}\,\alpha}{\psi e}}\exp\left(-\tfrac{1}{2}\tfrac{e^2y^2}{1+e}M\left(\alpha\right)\right)\tfrac{1}{\sqrt{2\pi}}\left(\tfrac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\tfrac{y^2}{2}\tfrac{e}{1+e}\right)dy.$$

Simplifying this further we get

$$= -\exp(aF) \int_{-\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\frac{\mathrm{ka}\,\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{ey^2}{1+e}\left(eM(\alpha)+1\right)\right) dy.$$

Notice that we have the density function of a $\left(\frac{1}{1+eM(\alpha)}\right)^{\frac{1}{2}} N\left(0, \left(\frac{1+e}{e(1+eM(\alpha))}\right)\right)$ distributed random variable in the above integral. Using the symmetry of the Normal distribution we have

$$E_i(U\,|e) = -2\exp(aF) \int_0^{\frac{\mathrm{ka}\,\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{ey^2}{1+e}\left(eM(\alpha)+1\right)\right) dy.$$

Substitute $s = \frac{ey^2}{1+e}\left(eM(\alpha)+1\right)$. Then, $ds = \frac{2ey}{1+e}\left(eM(\alpha)+1\right)dy$. For the limits of the integral: when $y = 0$ we have $s = 0$ and when $y = \frac{\mathrm{ka}\,\alpha}{\psi e}$ we have $s = \frac{(eM(\alpha)+1)}{1+e}\frac{(\mathrm{ka}\,\alpha)^2}{\psi^2 e}$. Using the above expression for s, we also get that $y = \pm\left(\frac{(1+e)s}{e(1+eM(\alpha))}\right)^{\frac{1}{2}}$. Since in the above integral y is strictly positive we can ignore the negative sign. Substituting this in the expectation we have

$$E_i(U\,|e)\Big|_{|y|\leq\frac{ka\alpha}{\psi e}} = -e^{aF}\left(\frac{1}{1+eM(\alpha)}\right)^{\frac{1}{2}} \int_0^{\frac{(eM(\alpha)+1)}{1+e}\frac{(ka\alpha)^2}{\psi^2 e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{s}{2}\right)\frac{1}{\sqrt{s}} ds.$$

The function inside the integral is the density function of a $\chi^2(1)$ random variable. Let Φ represent the cumulative distribution of a $\chi^2(1)\,variable.$ So, for this part we get

$$E_i(U\,|e)\Big|_{|y|\leq\frac{ka\alpha}{\psi e}} = -e^{aF}\left(\frac{1}{1+eM(\alpha)}\right)^{\frac{1}{2}} \Phi\left(\frac{(eM(\alpha)+1)}{1+e}\frac{(ka\alpha)^2}{\psi^2 e}\right). \quad \text{(a.9)}$$

<u>*For signals below the bound*</u> $(y < \frac{-ka\alpha}{\psi e})$

The expected utility of the investor in this region ignoring $-\exp(aF)$ is

$$= \int_{-\infty}^{-\frac{\mathrm{ka}\,\alpha}{\psi e}} \exp\left(\tfrac{ak(1-\alpha)}{1+e}\left(ye + \tfrac{1}{2}ak(1-\alpha)\right)\right) \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\tfrac{y^2}{2}\tfrac{e}{1+e}\right) dy,$$

$$= \int_{-\infty}^{-\frac{\mathrm{ka}\,\alpha}{\psi e}} \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\tfrac{-1}{2}\tfrac{e}{1+e}\left(y^2 - 2ayk(1-\alpha) - \tfrac{(ak(1-\alpha))^2}{e}\right)\right) dy.$$

Multiply and divide the integral by $\exp\left(-\frac{1}{2}\frac{e}{1+e}(ak(1-\alpha))^2\right)$. Then for the above equation we have

$$= \int_{-\infty}^{-\frac{\mathrm{ka}\,\alpha}{\psi e}} \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\tfrac{-1}{2}\tfrac{e}{1+e}\left((y-ak(1-\alpha))^2 - \tfrac{1+e}{e}(ak(1-\alpha))^2\right)\right) dy,$$

$$= \exp\left(\tfrac{(ak(1-\alpha))^2}{2}\right) \int_{-\infty}^{-\frac{\mathrm{ka}\,\alpha}{\psi e}} \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\tfrac{-1}{2}\tfrac{e}{1+e}(y-ak(1-\alpha))^2\right) dy.$$

Now make the following substitution $s = \frac{e}{1+e}(y-ak(1-\alpha))^2$. Then $ds = \frac{2e}{1+e}(y-ak(1-\alpha))\,dy$. Based on the above equation $y = \pm\left(\frac{1+e}{e}s\right)^{\frac{1}{2}} + ak(1-\alpha)$. Since we are strictly restricting ourselves to the real line it has to be that $s > 0$. Note, in this case using the negative part of the expression of y is the only sensible thing to do since it is the only thing that will work when $y = -\infty$. For the limits of integral, when $y = -\infty$ $s = \infty$ and when $y = -\frac{ka\alpha}{\psi e}$ $s = \frac{e}{1+e}\left(\frac{ka\alpha}{\psi e} + ka(1-\alpha)\right)^2$. It can be seen that $\frac{e}{1+e}\left(\frac{ka\alpha}{\psi e} + ka(1-\alpha)\right)^2$ can be expressed as $\frac{(ka\alpha)^2}{\psi^2 e}\frac{(1+em(\alpha))^2}{1+e}$. Making these substitutions we get the following

$$E_i(U|e)\Big|_{y<\frac{-ka\alpha}{\psi e}} = \frac{1}{2}\exp\left(\frac{(ak(1-\alpha))^2}{2}\right) \int_{\frac{(ka\alpha)^2}{\psi^2 e}\frac{(1+em(\alpha))^2}{1+e}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-s}{2}\right) \frac{(-ds)}{\sqrt{s}}.$$

This gives the expected utility for this region

$$E_i\left(U\left|e\right.\right)\Big|_{y<\frac{-ka\alpha}{\psi e}} = -\frac{e^{aF}}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right). \tag{a.10}$$

<u>*For signals above the bound*</u> $(y > \frac{ka\alpha}{\psi e})$

The expected utility of the investor in this region ignoring $-\exp\left(aF\right)$ is

$$= \int_{\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\infty} \exp\left(\tfrac{-ak(1-\alpha)}{1+e}\left(ye-\tfrac{1}{2}ak\left(1-\alpha\right)\right)\right)\tfrac{1}{\sqrt{2\pi}}\left(\tfrac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\tfrac{y^2}{2}\tfrac{e}{1+e}\right)dy$$

$$= \int_{\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\infty} \tfrac{1}{\sqrt{2\pi}}\left(\tfrac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\tfrac{1}{2}\tfrac{e}{1+e}\left(y^2+2\,\mathrm{ya}\,k\left(1-\alpha\right)-\tfrac{(ak(1-\alpha))^2}{e}\right)\right)dy.$$

Multiplying and dividing by $\exp\left(-\frac{1}{2}\frac{e}{1+e}\left(ak\left(1-\alpha\right)\right)^2\right)$ we get that above is

$$= \exp\left(\tfrac{(ak(1-\alpha))^2}{2}\right)\int_{\frac{\mathrm{ka}\,\alpha}{\psi e}}^{\infty}\tfrac{1}{\sqrt{2\pi}}\left(\tfrac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\tfrac{1}{2}\tfrac{e}{1+e}\left(y+ak\left(1-\alpha\right)\right)^2\right)dy.$$

Now make the following substitution $s = \frac{e}{1+e}\left(y+ak\left(1-\alpha\right)\right)^2$. Then $ds = \frac{2e}{1+e}\left(y+ak\left(1-\alpha\right)\right)dy$. Based on the above equation $y = \pm\left(\frac{1+e}{e}s\right)^{\frac{1}{2}} - ak\left(1-\alpha\right)$. In this case using the positive part of the expression of y is the only sensible thing to do. For the limits of integral, when $y=\infty$ $s=\infty$ and when $y=\frac{ka\alpha}{\psi e}$, $s=\frac{(ka\alpha)^2}{\psi^2 e}\frac{(1+em(\alpha))^2}{1+e}$. Substituting these in the equation for expected utility we have

$$E_i\left(U\left|e\right.\right)\Big|_{y>\frac{ka\alpha}{\psi e}} = \frac{1}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\int_{\frac{(ka\alpha)^2}{\psi^2 e}\frac{(1+em(\alpha))^2}{1+e}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{s}{2}\right)\frac{1}{2\sqrt{s}}ds.$$

This gives the expected utility for this region

$$E_i\left(U\left|e\right.\right)=-\frac{e^{aF}}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right) \tag{a.11}$$

Adding these three parts up, equation(a.9), equation(a.10), and equation(a.11), we have the following expression for the overall unconditional expected utility

$$E\left(U_i\left|e\right.\right)=-\exp\left(aF\right)f\left(\alpha,e\right),$$

where

$$\begin{aligned}f\left(\alpha,e\right)=&\left(\frac{1}{1+eM\left(\alpha\right)}\right)^{\frac{1}{2}}\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(eM\left(\alpha\right)+1\right)}{1+e}\right)+\\&\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right).\end{aligned}$$